Evolution of population genetics

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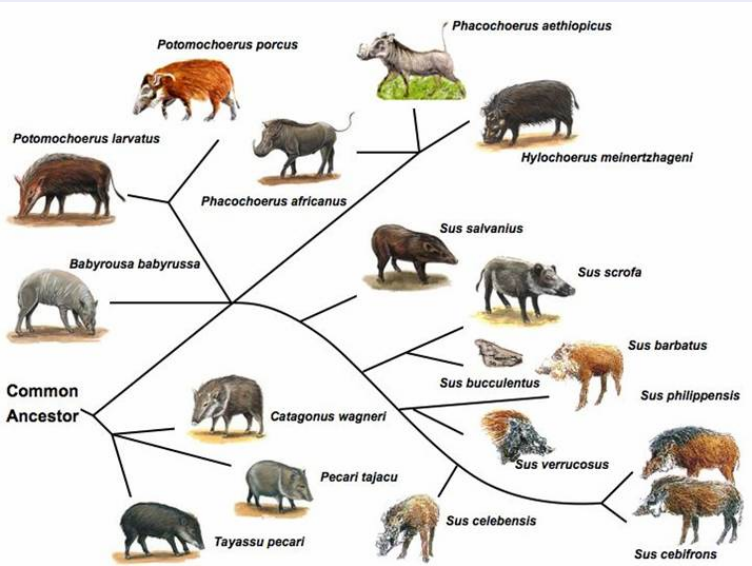
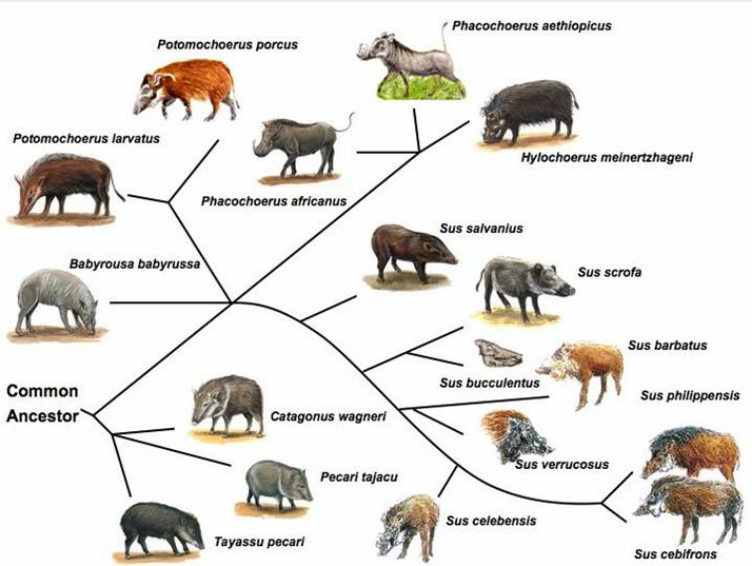
March 11, 2019

Modeling the evolution of genes



Probability in mathematical biology

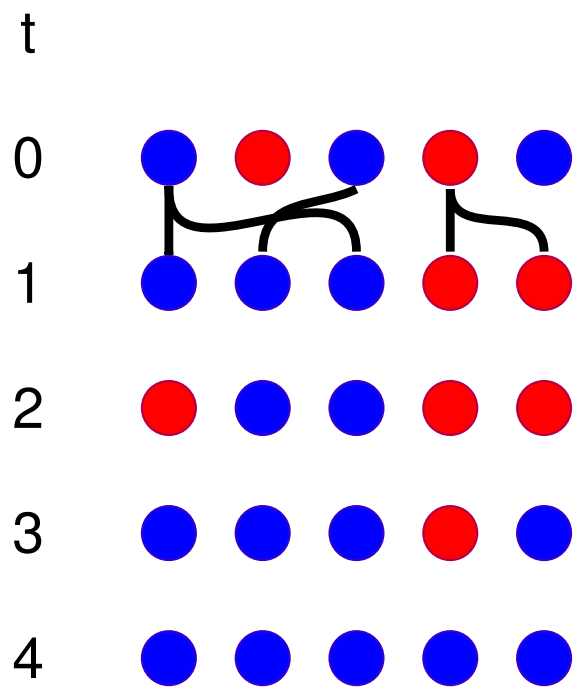
Modeling the evolution of genes



Competing types of one species

Wright-Fisher model

- N individuals
- Individuals of 2 types, ● and ●
- $X_t = \#$ ● individuals at time t



Wright-Fisher model

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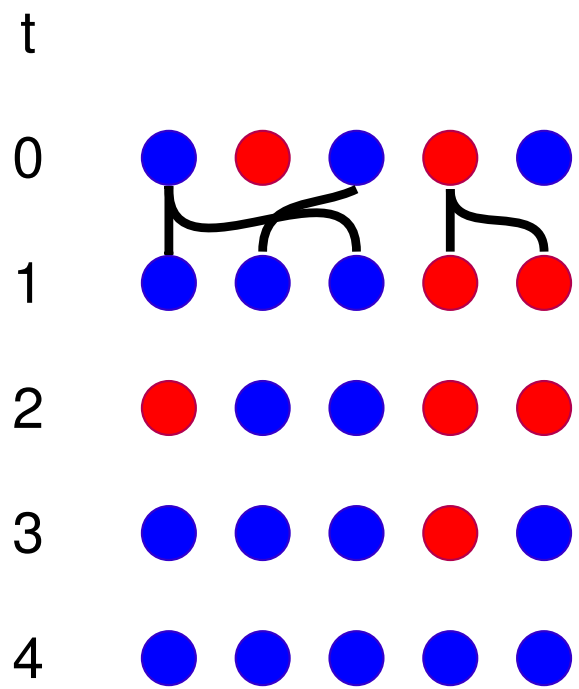
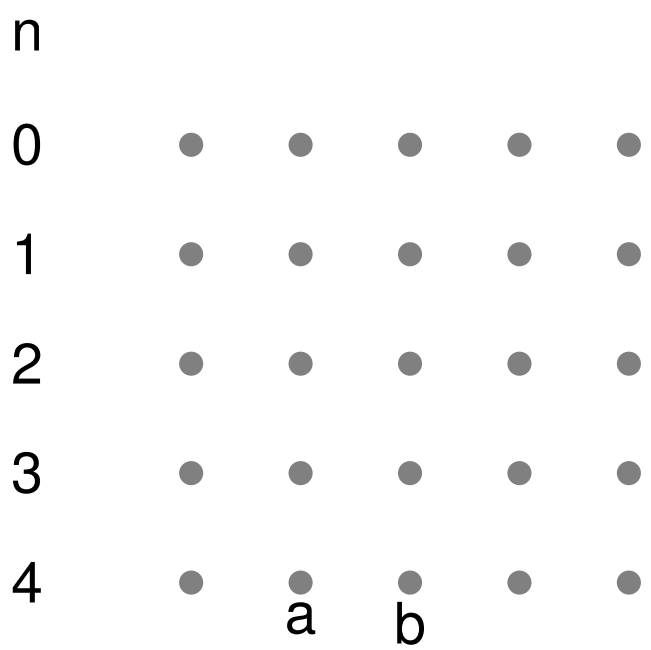


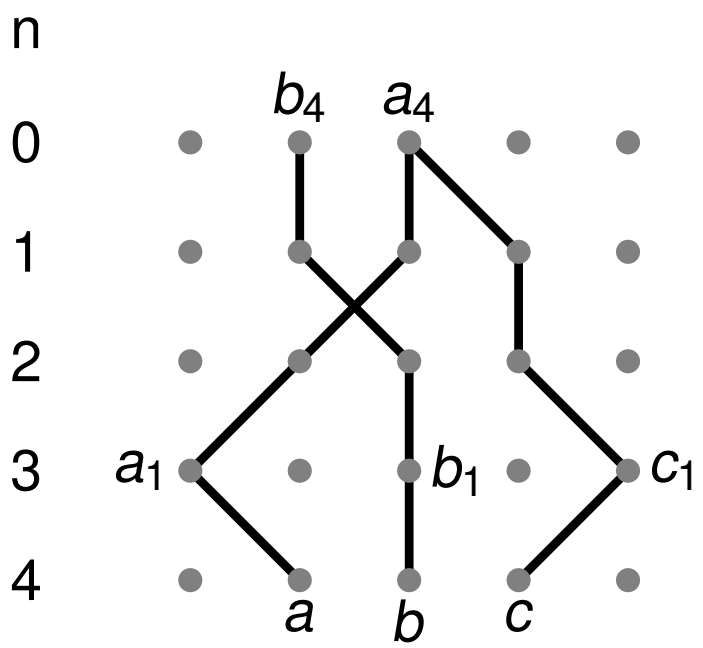
- If $X_t = 0$ or $X_t = N$ the system will not change anymore!

Wright-Fisher model

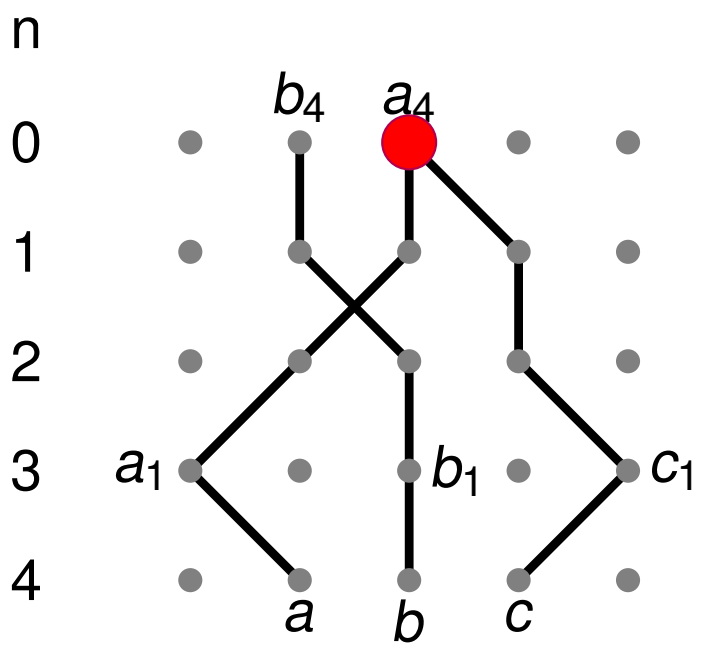


Suppose we have N individuals and we know X_0. What is the probability that two individuals a and b picked uniformly at random in generation n, are of the same type?

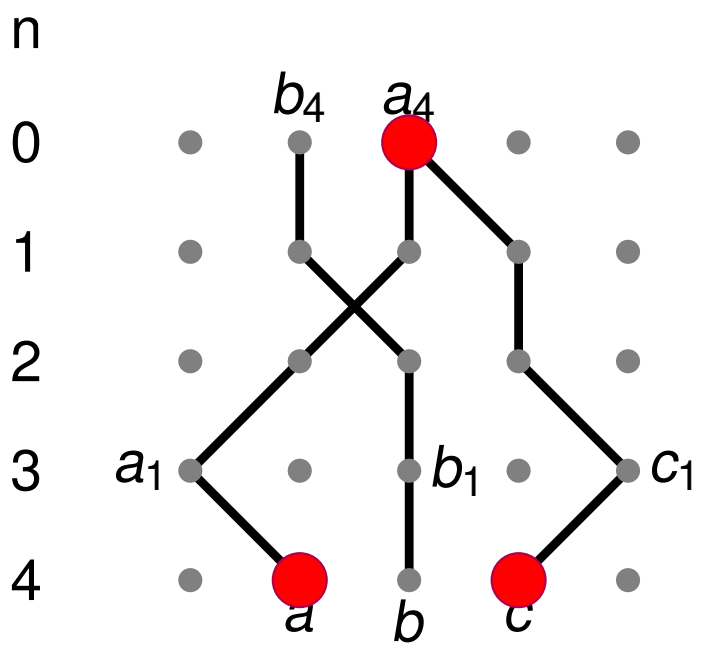
Ancestral lines



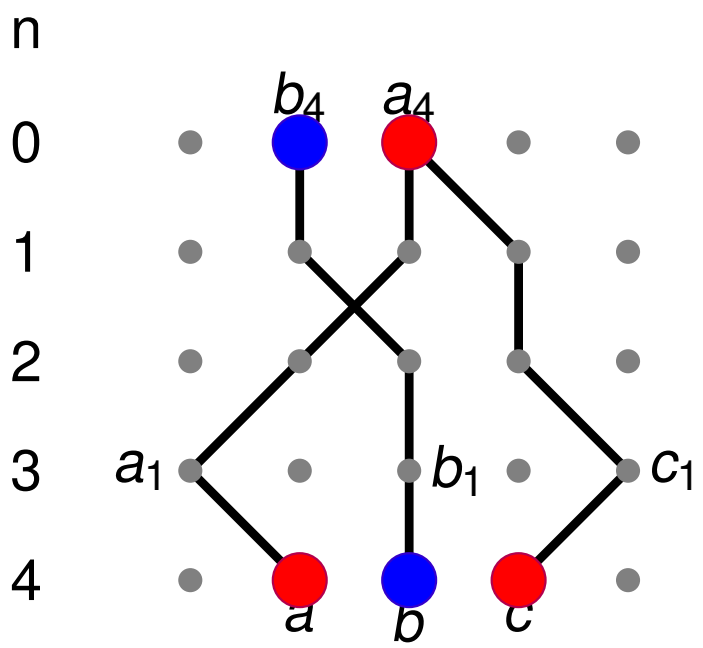
Ancestral lines



Ancestral lines



Ancestral lines



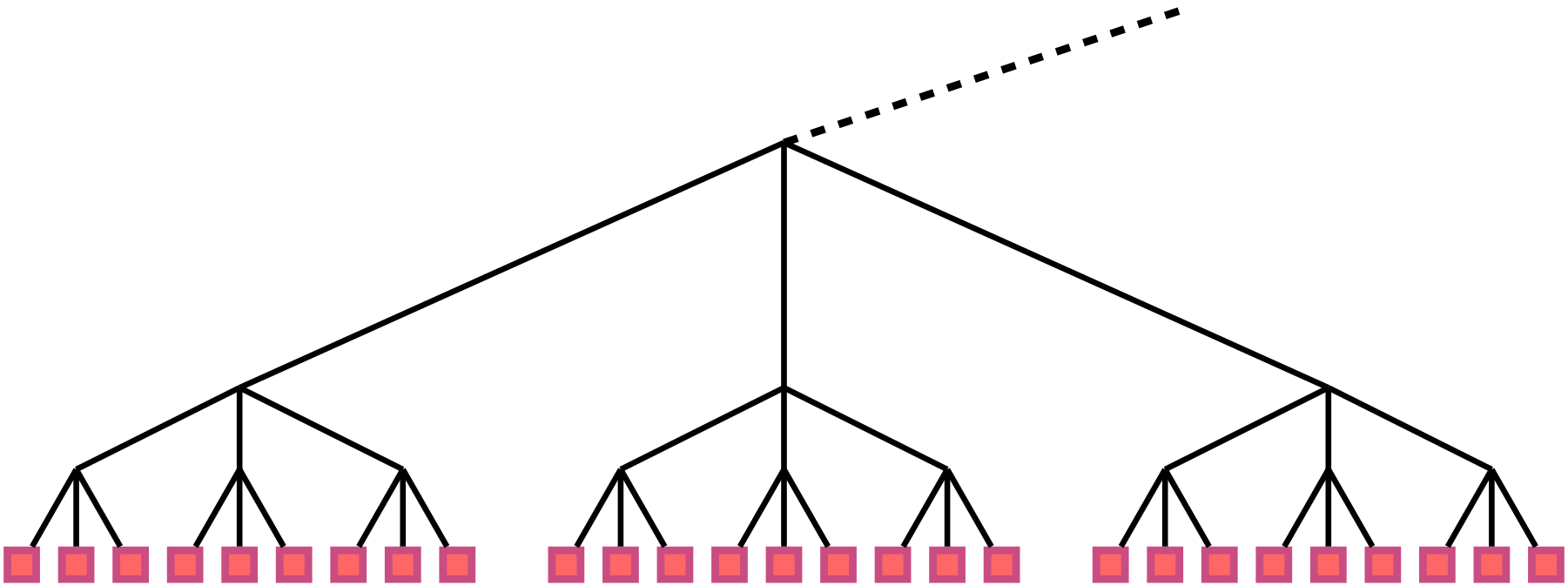
Continuum limit

- Single colony model
  - 2 types, ● and ●
  - X_t=# ● individuals in active population
- Continuum limit
  - Speed up time by the size of the active population
  - Let the size of the population grow to ∞

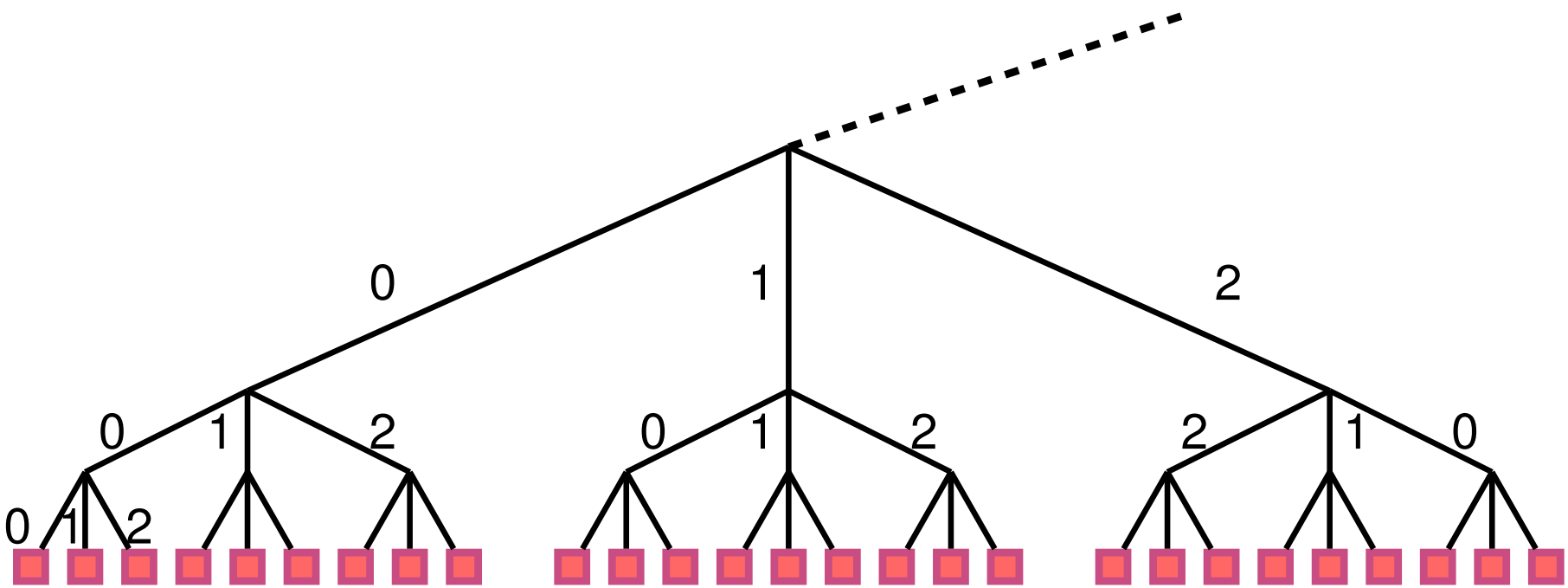
$$\lim_{N\to\infty} \frac{X_N(Nt)}{N} = X(t)$$

$$dX_t = \overbrace{\sqrt{X_t(1-X_t)}dW_t}^{resampling}$$

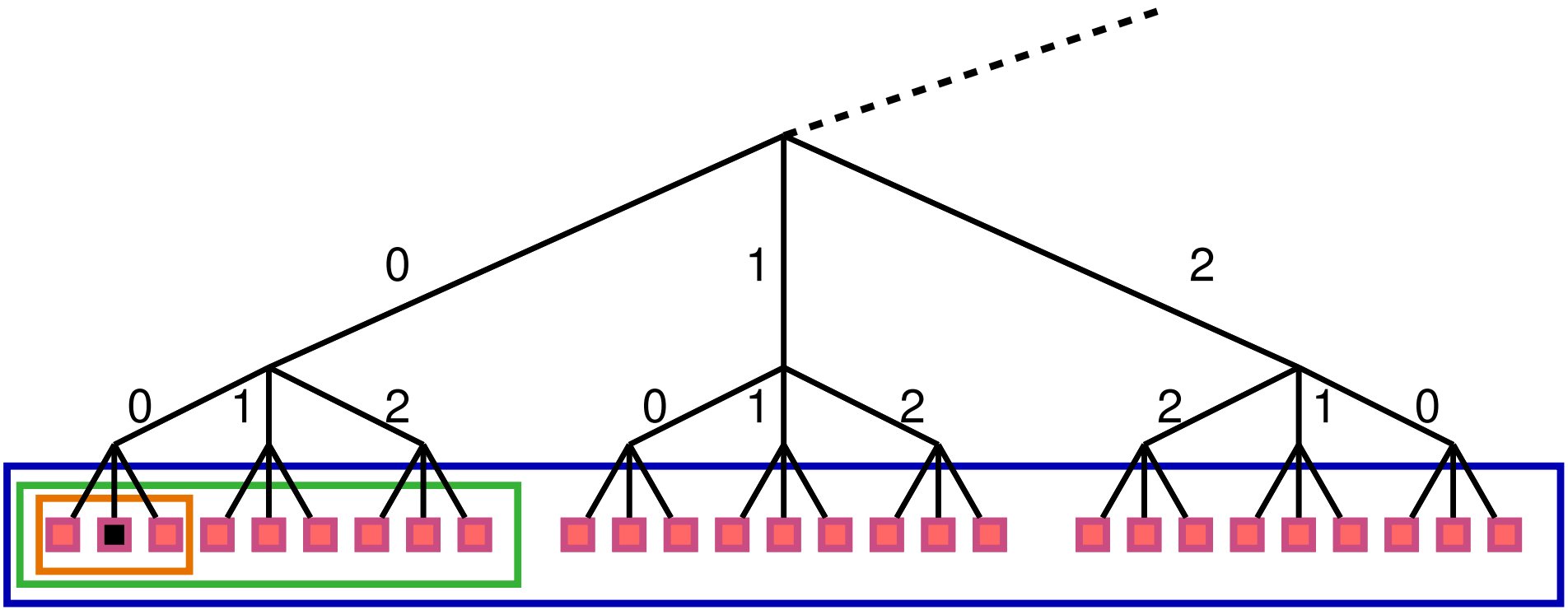
Hierarchical lattice Ω_3



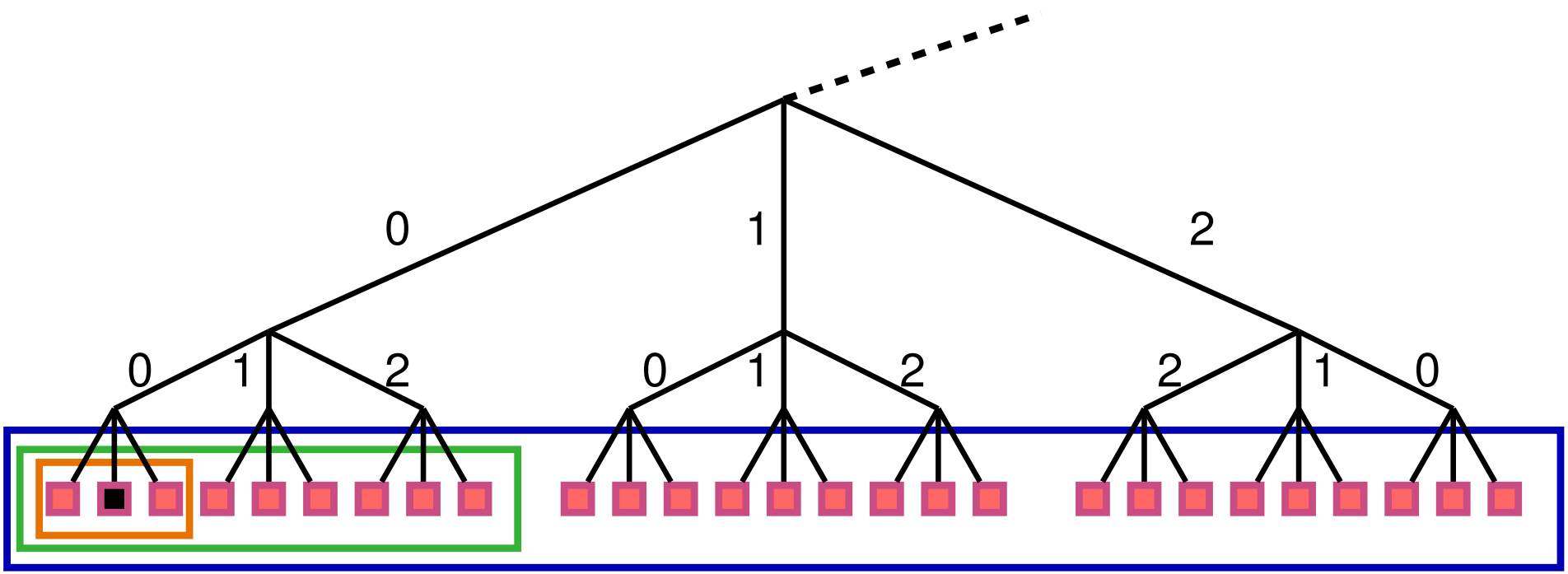
Hierarchical lattice Ω_3



Hierarchical lattice Ω_3



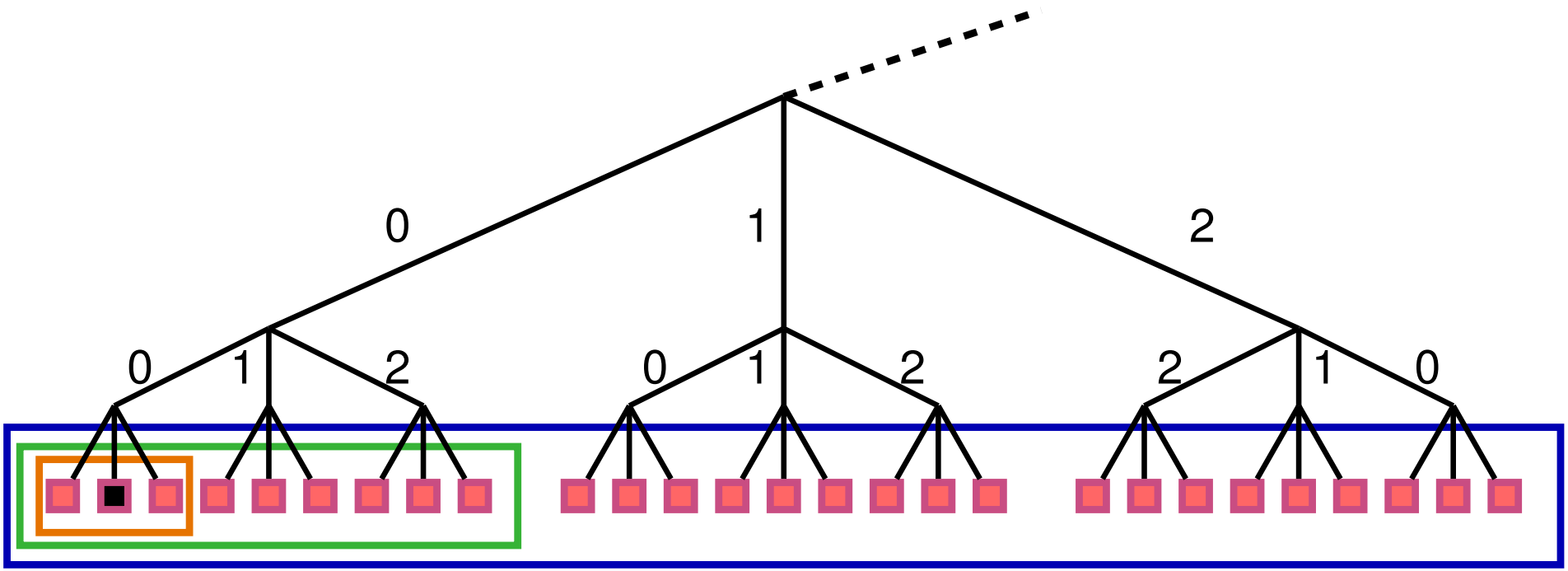
Hierarchical lattice Ω_3



$$\Omega_3 = \left\{ \xi = (\xi_n)_{n \in \mathbb{N}_0} : \xi_n \in \{0,1,2\}, \sum_{n \in \mathbb{N}_0} \xi_n < \infty \right\}$$

$$d(\xi,\eta) = \min\{m \in \mathbb{N}_0 : \xi_n = \eta_n \ \forall n \geq m\}$$

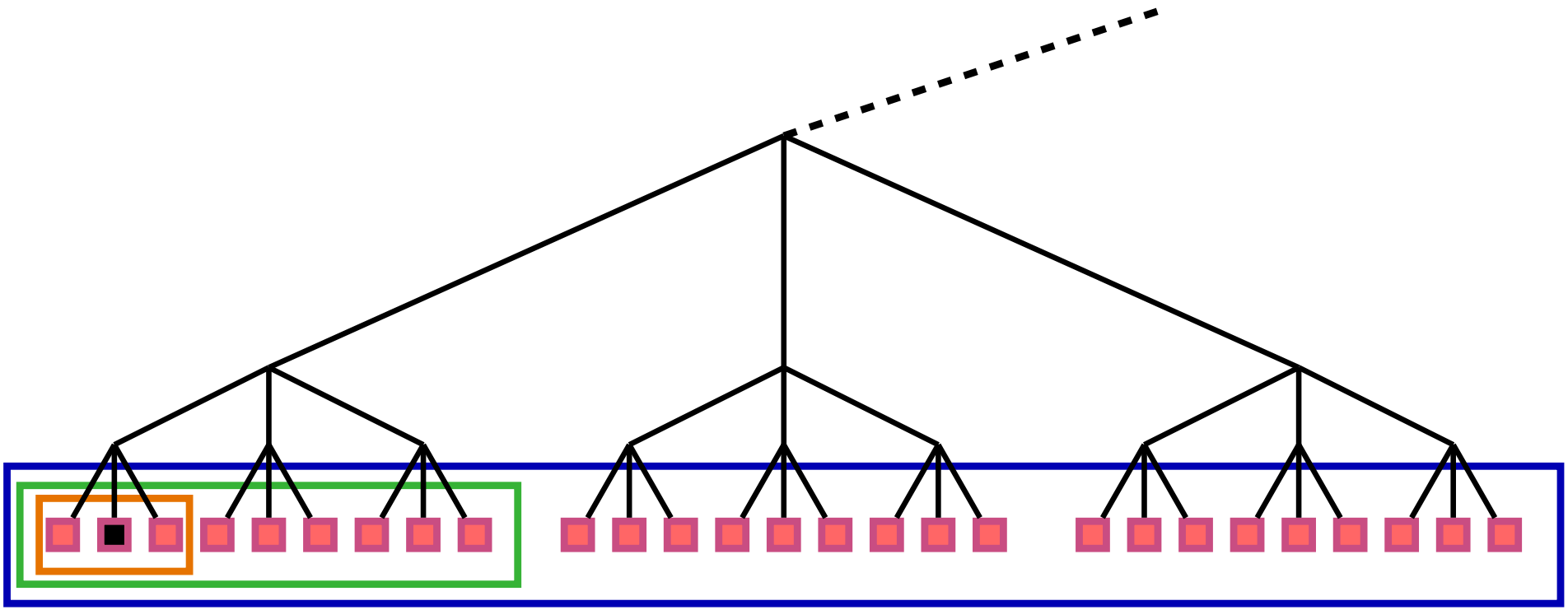
Hierarchical lattice Ω_3 with migration



Migration kernel

$$a(\xi,\eta) = \sum_{k \geq d(\xi,\eta)} \frac{1}{3^{k-1}} \frac{1}{3^k}, \qquad \xi, \eta \text{ sites in } \Omega_3$$

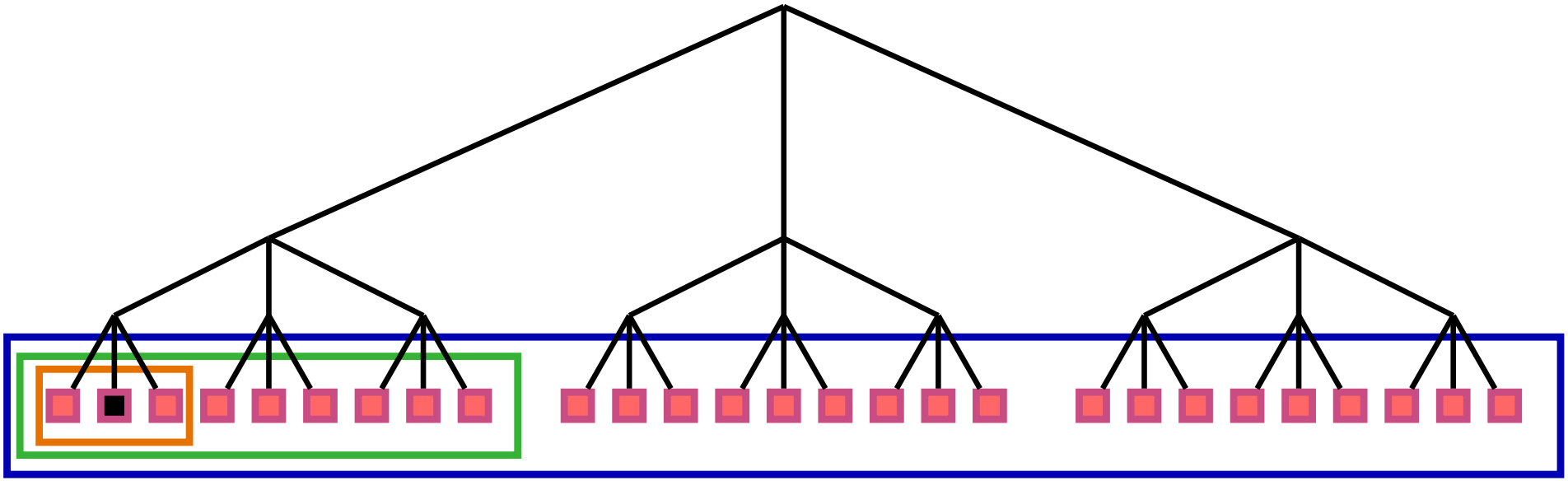
Evolution single colony



- System of SDE

$$dX_\xi(t) = \overbrace{\sum_{\eta\in\Omega_3} a(\xi,\eta)(X_\eta(t) - X_\xi(t))dt}^{migration} + \overbrace{\sqrt{X_\xi(t)(1-X_\xi(t))}dW_\xi(t)}^{resampling}$$

Block averages

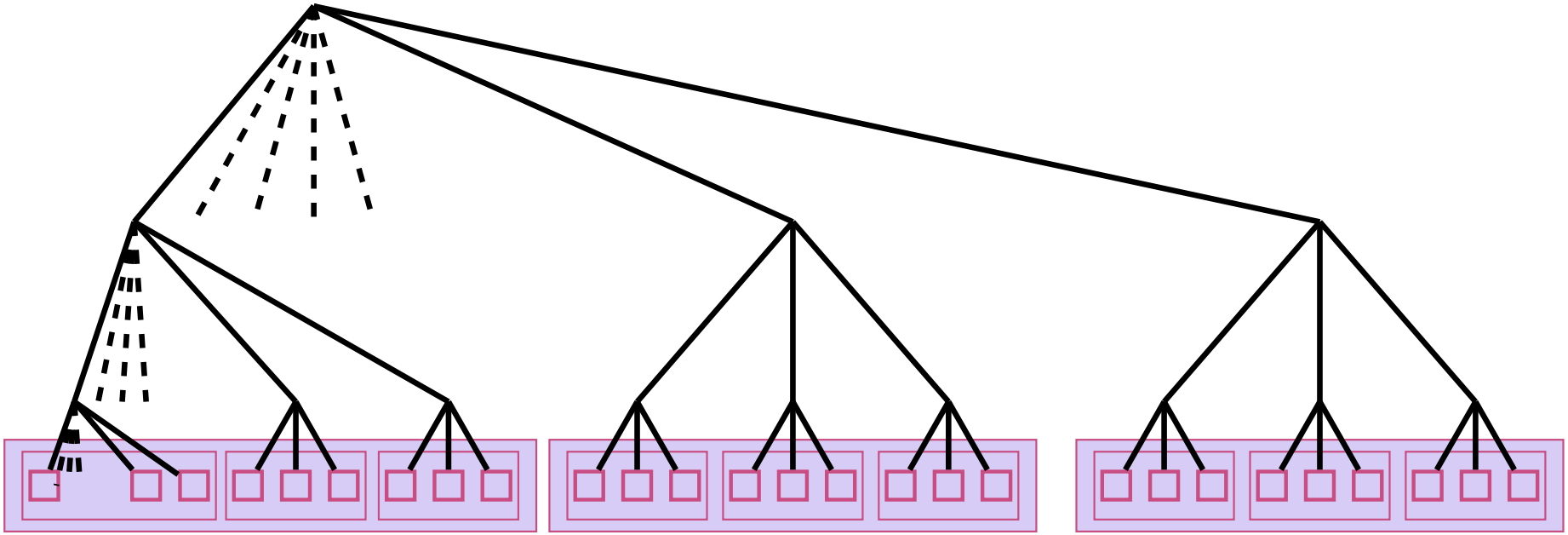


- Block averages

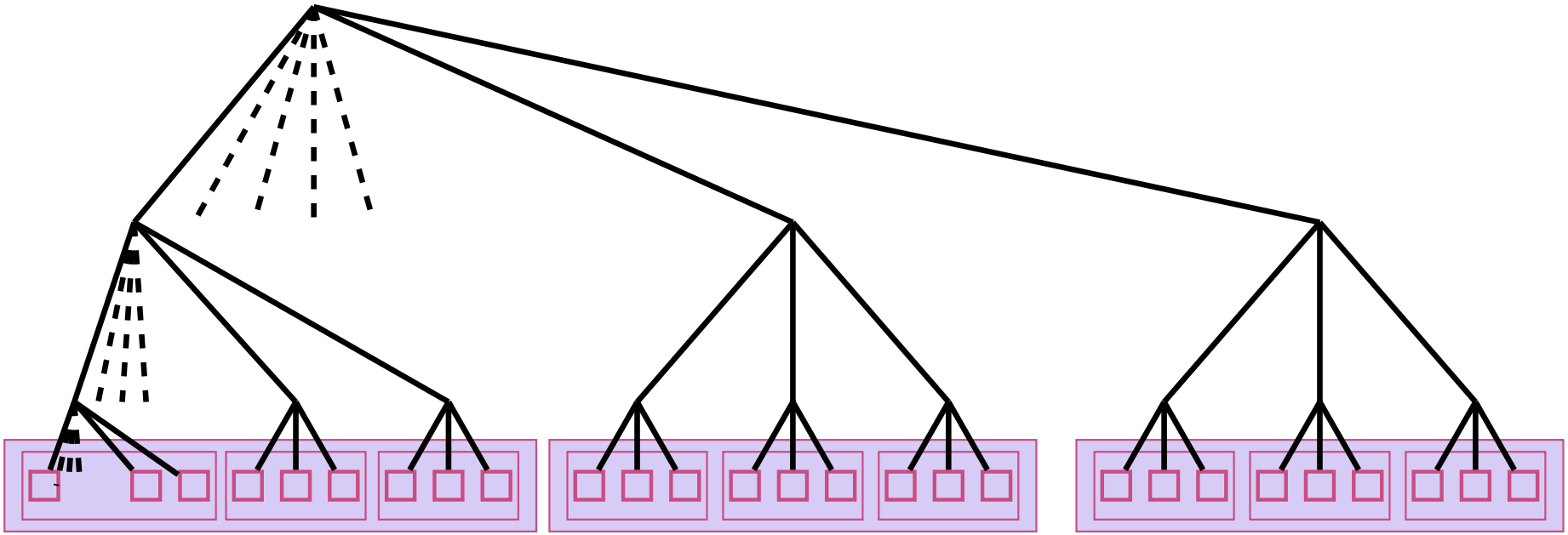
$$X_k^{[3]}(t) = \frac{1}{3^k} \sum_{\xi \in B_k(0)} X_\xi(3^k t)$$

- What are the dynamics of the block averages, i.e. the streets, the cities, the countries and so on?

Hierarchical lattice Ω_M



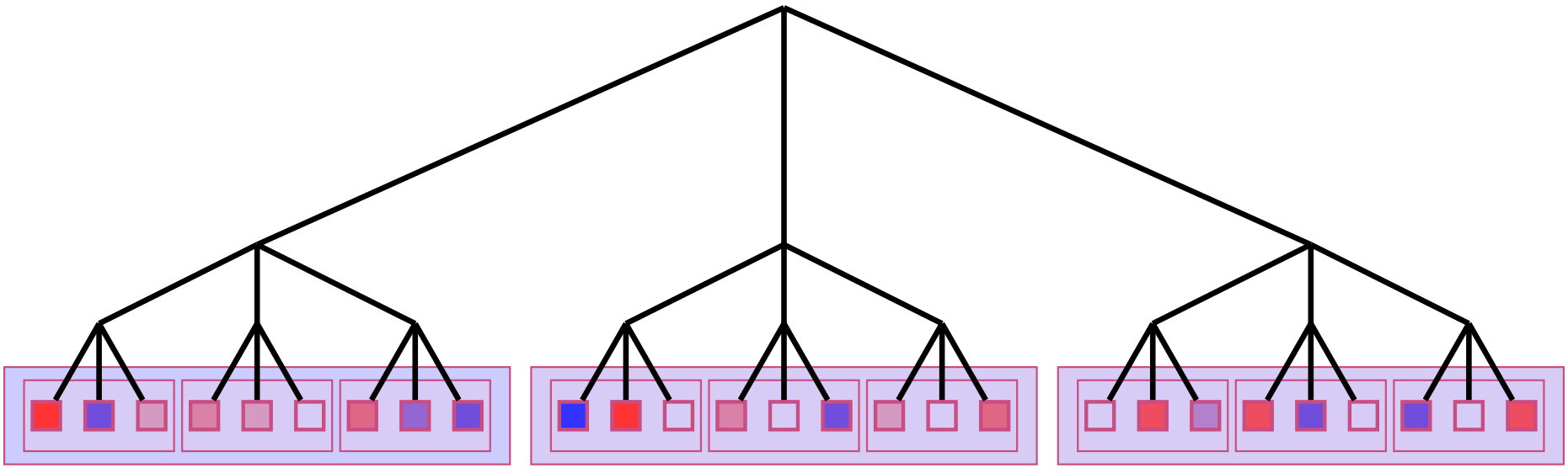
Hierarchical lattice Ω_M



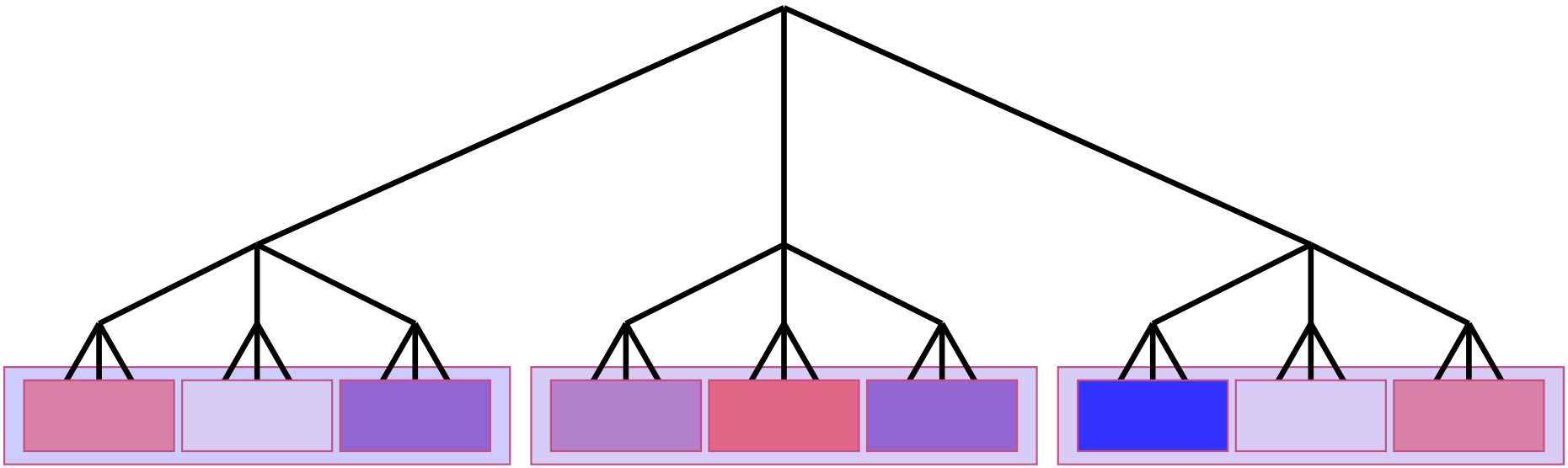
- Migration kernel

$$a(\xi,\eta) = \sum_{k \geq d(\xi,\eta)} \frac{1}{M^{k-1}} \frac{1}{M^k}, \qquad \xi, \eta \text{ sites in } \Omega_M$$

Timescale t



Timescale *Mt*



Timescale $M^2 t$