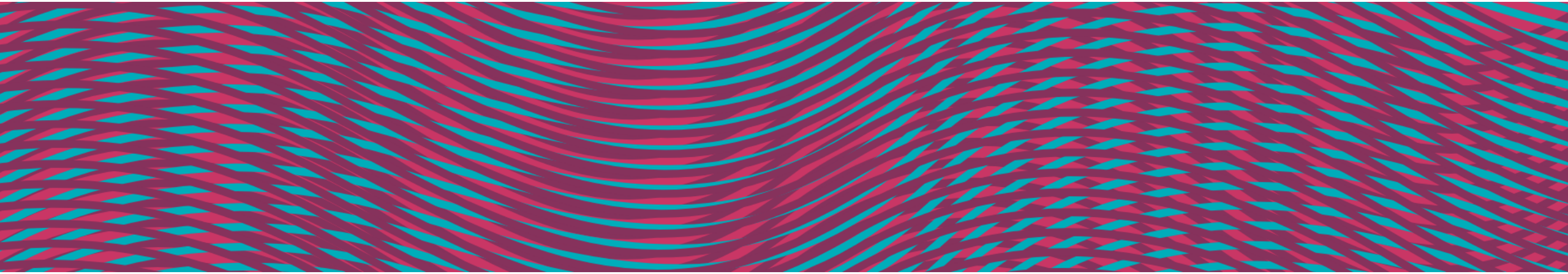
NETWORKS goes to school 2024
University of Amsterdam

Thursday 14th of November

Nicos Starreveld, Koen van den Berk, Artem Tsikiridis



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TU/e

UNIVERSITEIT VAN AMSTERDAM

CWI

10-11: Introduction on Graphs, Flows, Game theory and Nash equilibria

11-12:30: Artem Part 1: Traffic models, a mathematical model of selfish driving, Nash flow.

12:30-13:30: Lunch

13:30-14:30: Artem part 2: Optimal flows, price of anarchy and tolls.

14:30-15:00: break

15:00-16:00: Artem part 3: Braess' paradox.

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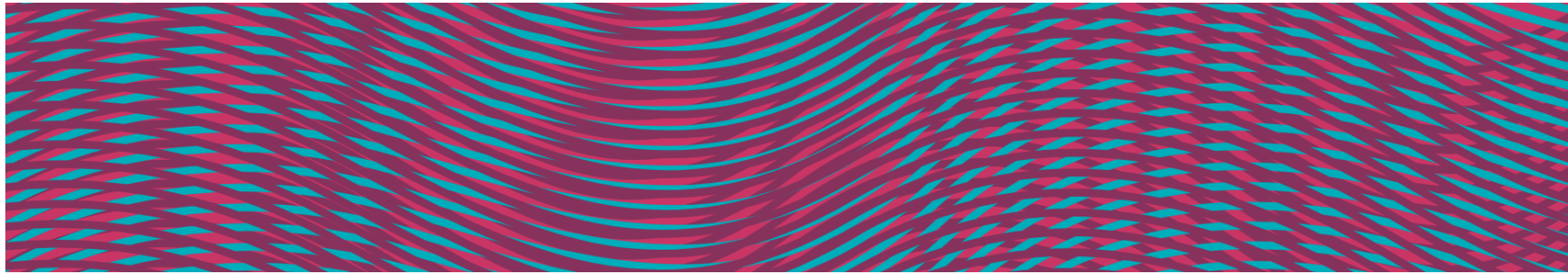

NETWORKS

Many real-world phenomena can be phrased in terms of **networks**.

A network consists of **objects with connections** between them, and possibly also properties of the objects and their connections.

- ✓ Road networks and traffic congestion.
- ✓ Electricity networks and electricity flows.
- ✓ Network of contacts.
- ✓ 5G telecommunication networks.

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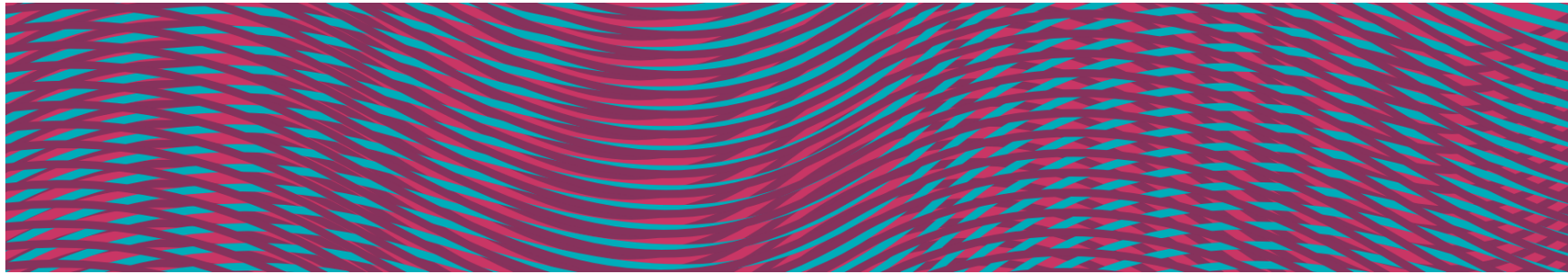

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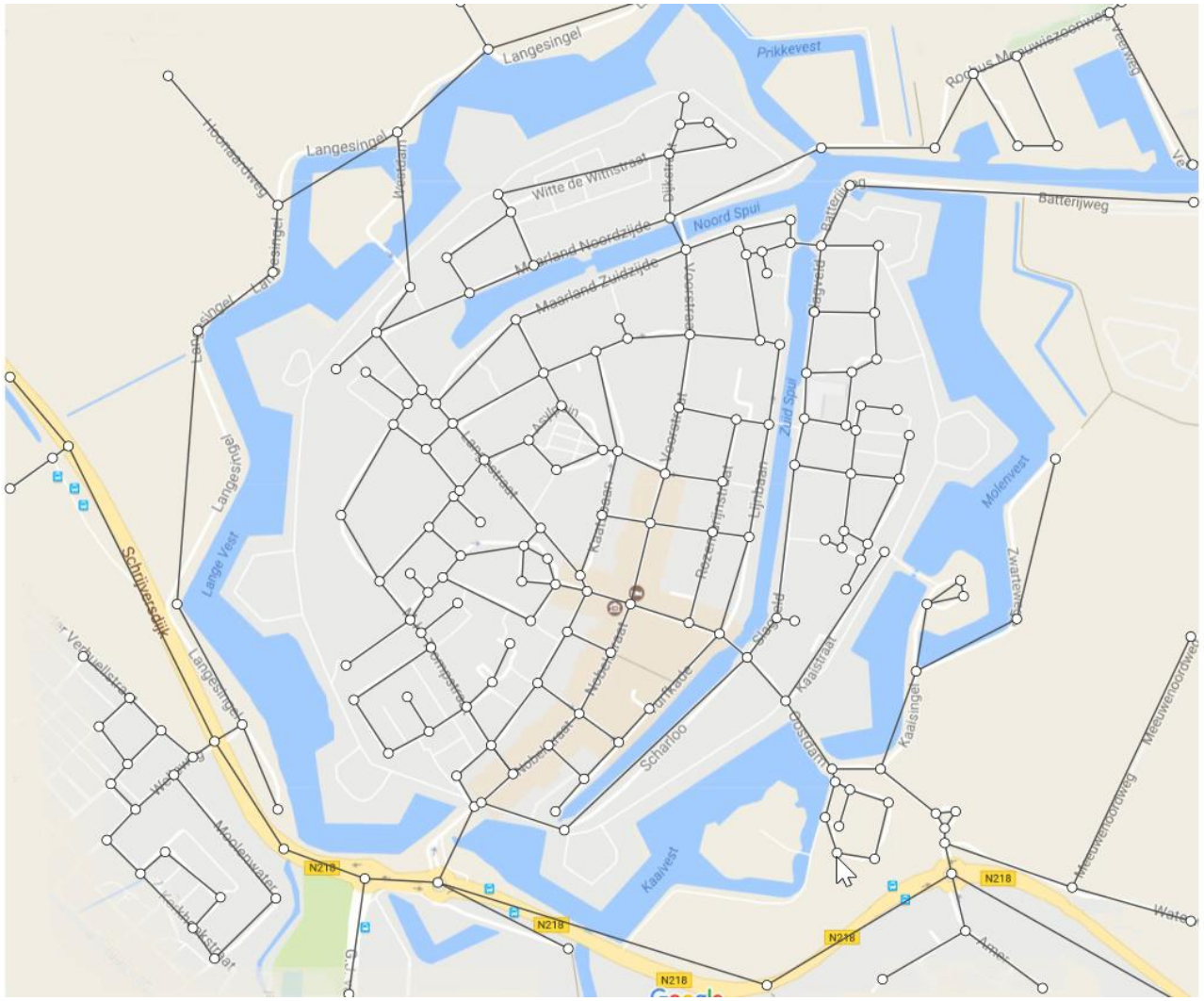

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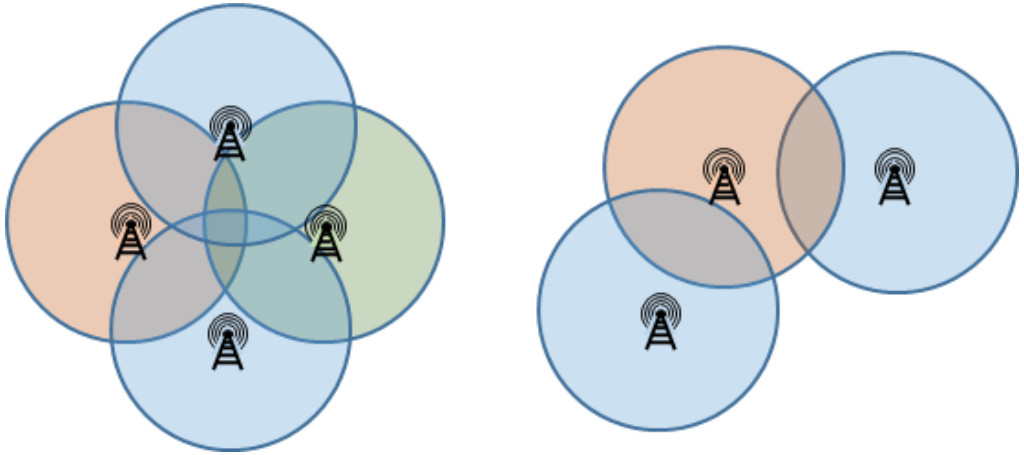

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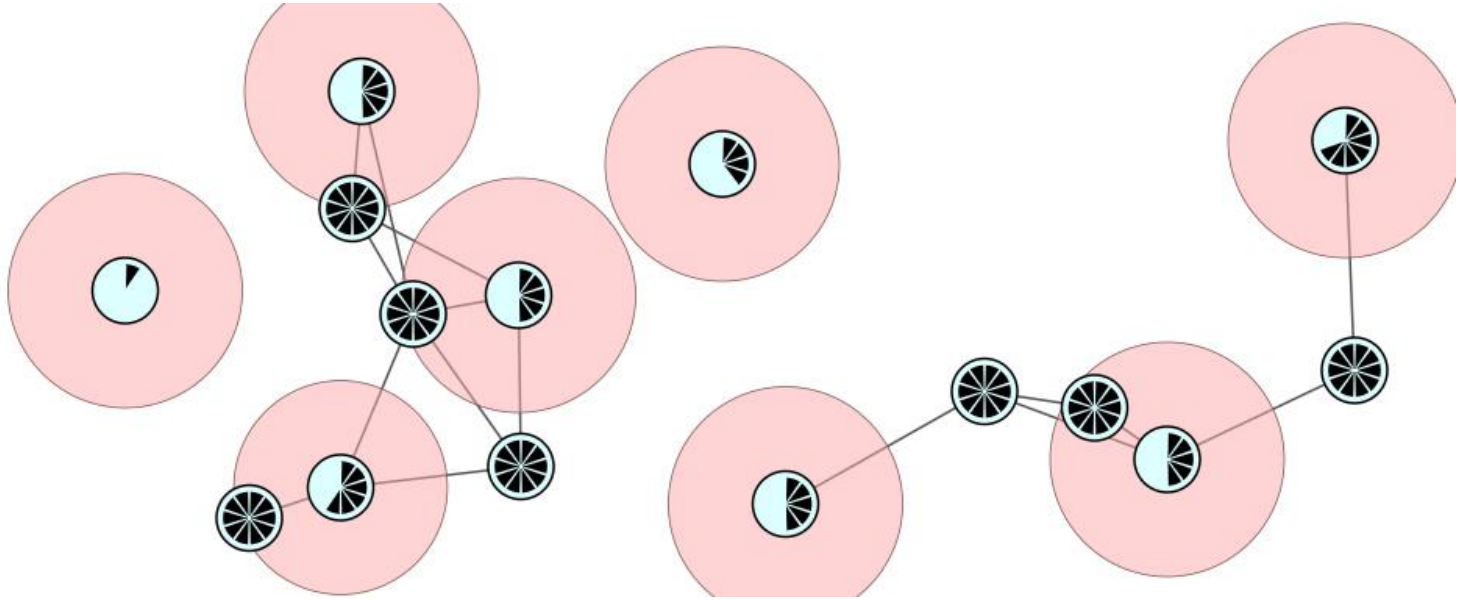

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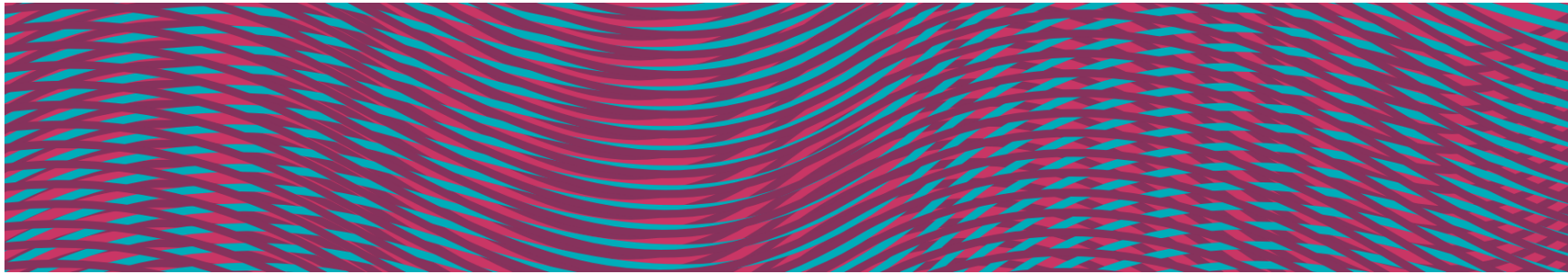

NETWORKS

(a) KPN.

(b) Vodafone.

(c) Odido.

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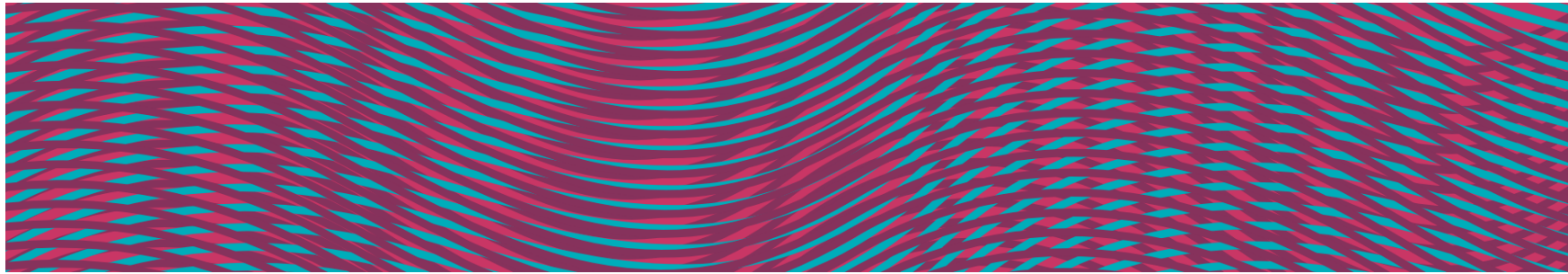

NETWORKS

Mathematics play an important role in understanding and developing networks.

Fundamental, creative, challenging, applied!

- ✓ Grath theory (the infrastructure)
- ✓ Game theory (the decisions)
- ✓ Probability theory (the randomness) **(TUESDAY)**
- ✓ Algorithms (how the network works)
- ✓ Queueing theory (how to optimize what the network is doing) **(TUESDAY)**

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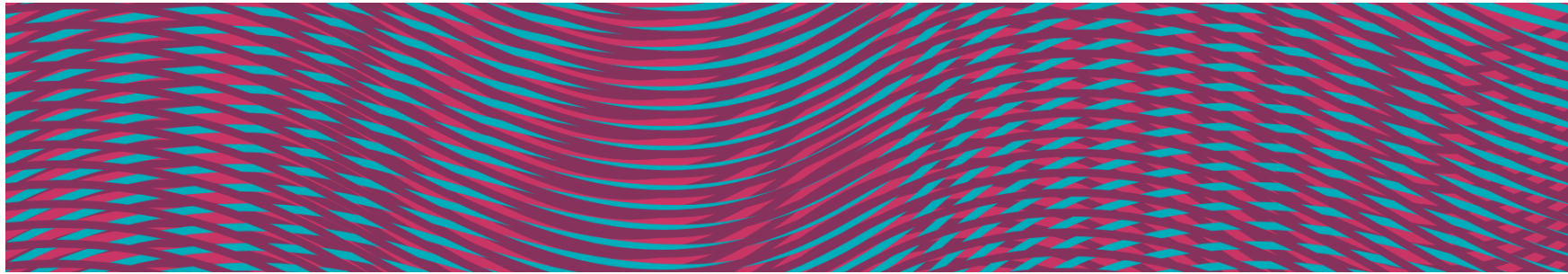


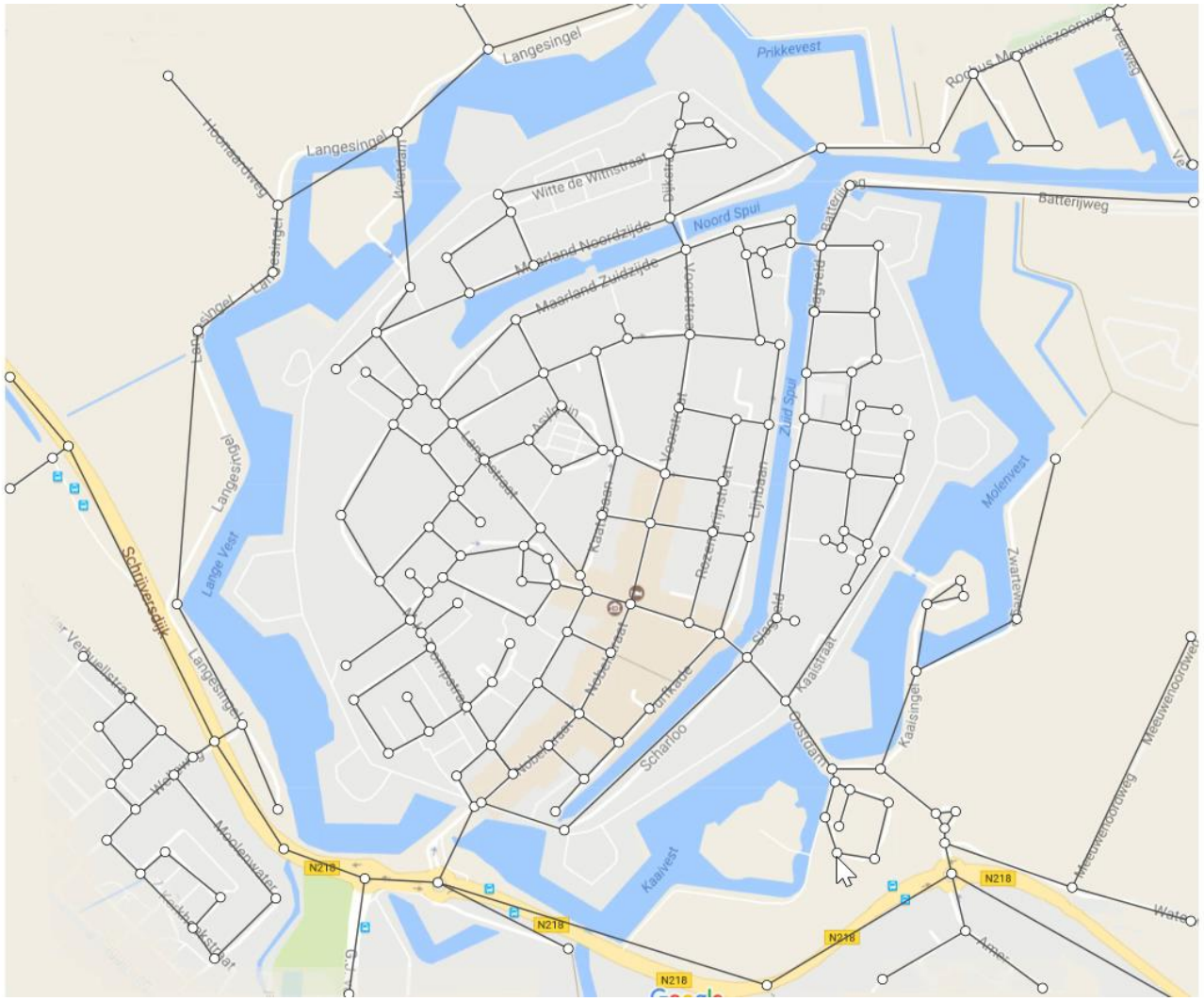
NETWORKS

Many real-world phenomena can be phrased in terms of networks.

Grath theory (the infrastructure)

Algorithms (how the network works)

An algorithm is a step-by-step procedure to perform a given task. Algorithms can be executed by computers, but also by persons.



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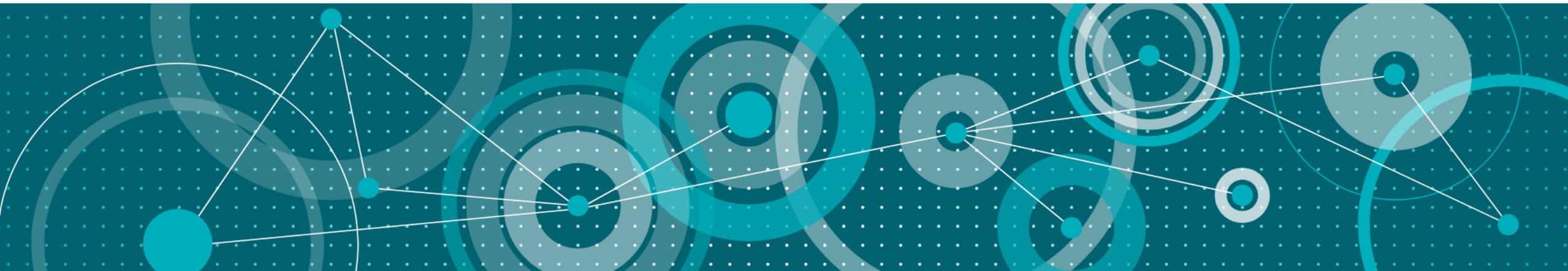
Graph theory

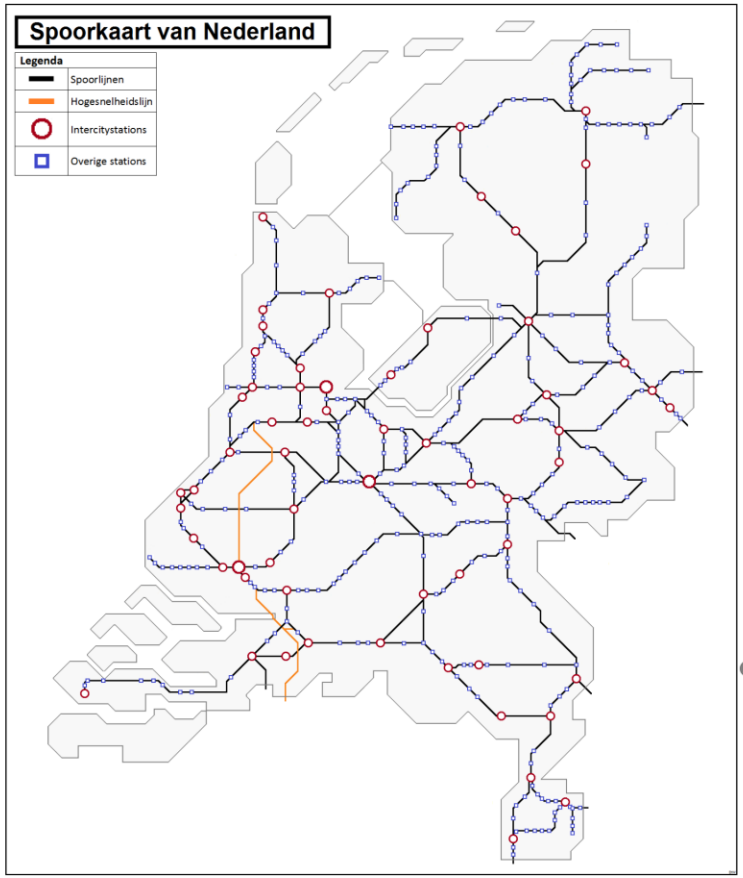
The structure of the network

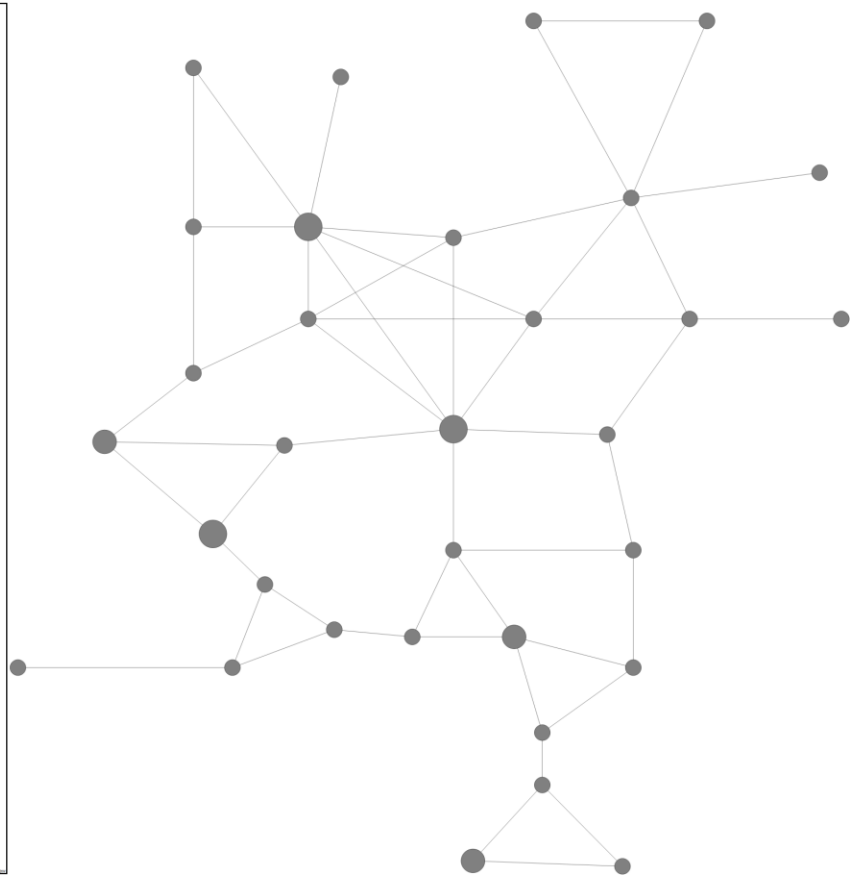


Graph theory

A network consists of objects with connections between them.

In mathematics a network is called a ***graph***, and objects are called ***vertices*** (**or *nodes***) and the connections are called ***edges***.





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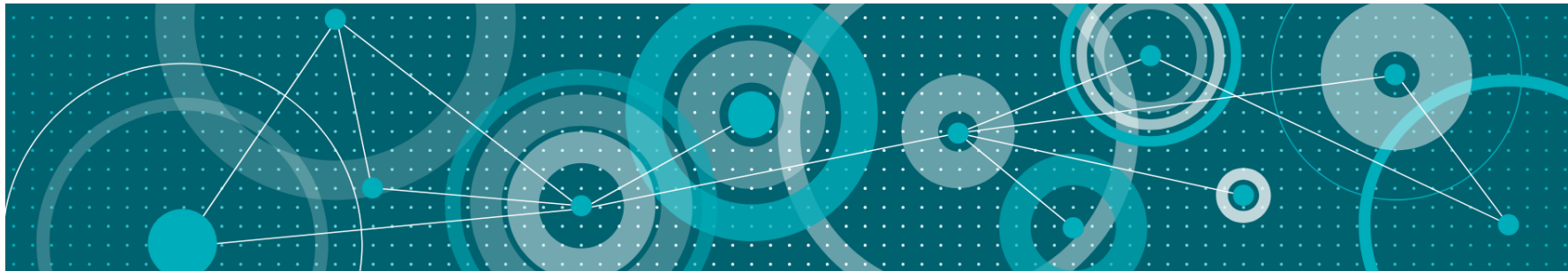


Graph theory

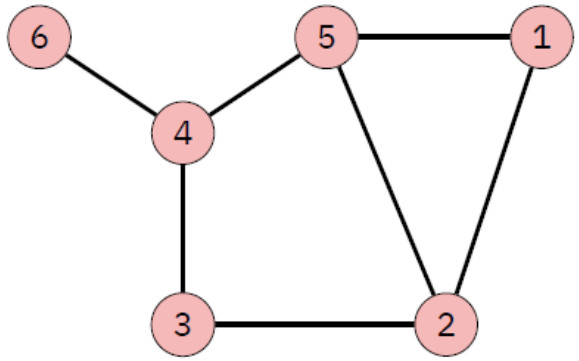
An (undirected) graph is a pair
G = (V, E), where

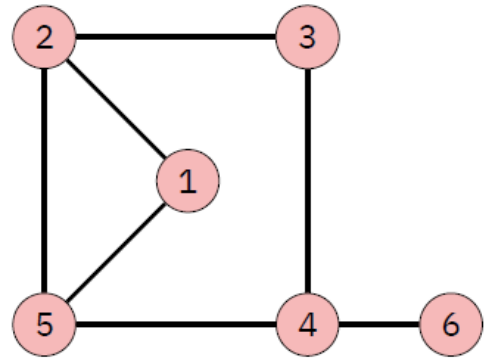
- V is the set of nodes or vertices;
- E is the set of edges, connecting the nodes.

Exercise 1. Draw the graph

V = {1; 2; 3; 4; 5; 6};
E = {{1; 2}; {1; 5}; {2; 3}; {2; 5}; {3; 4}; {4; 5}; {4; 6}}

Exercise 2: Translate to mathematical notation the following graphs.





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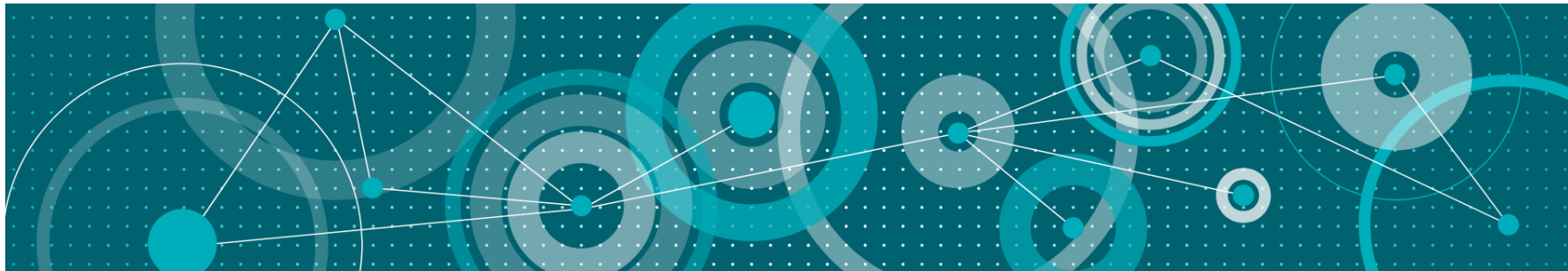


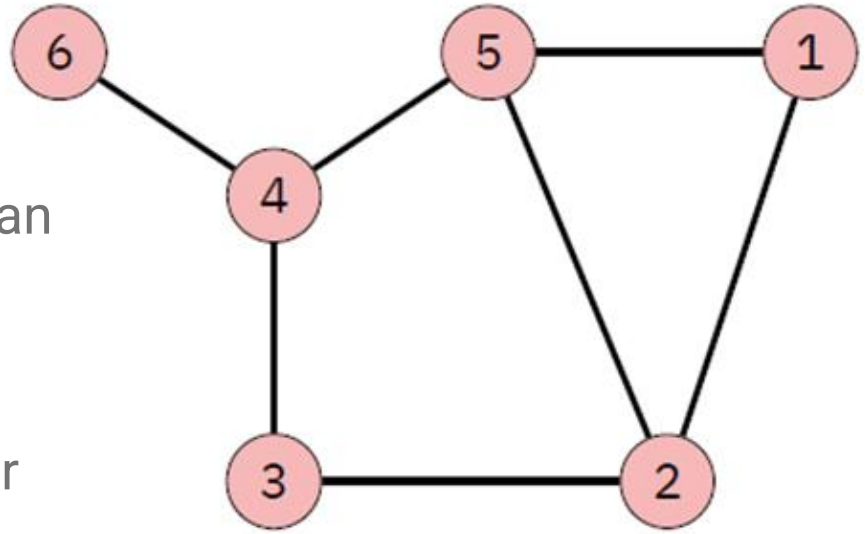
Graph theory – (shortest) paths

A path between two nodes in a graph, say v and w, is a sequence of edges which joins a sequence of nodes from v to w.

The sequence 6 → 4 → 3 → 2 forms a path from 6 to 2.

The sequence 6 → 4 → 3 → 1 is not a path since {3; 1} is not an edge in the graph.

A **shortest path** between two nodes is a path using the least amount of edges. The shortest path from node 6 to node 1 for example has length 3.



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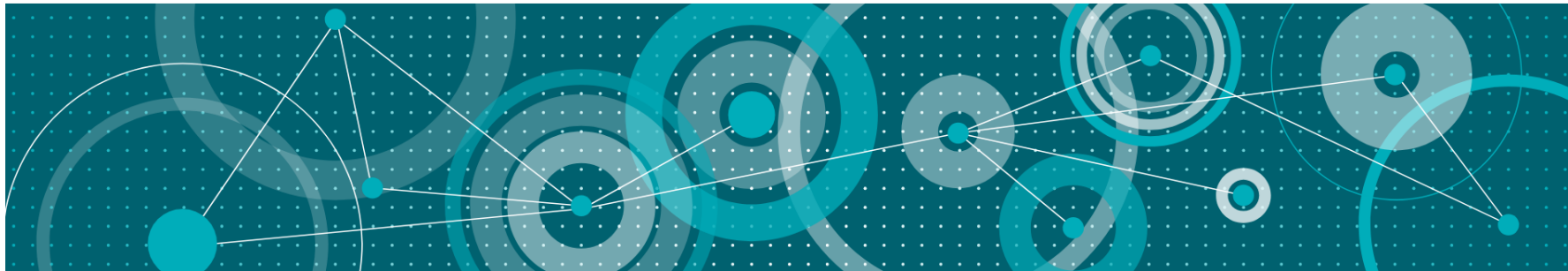
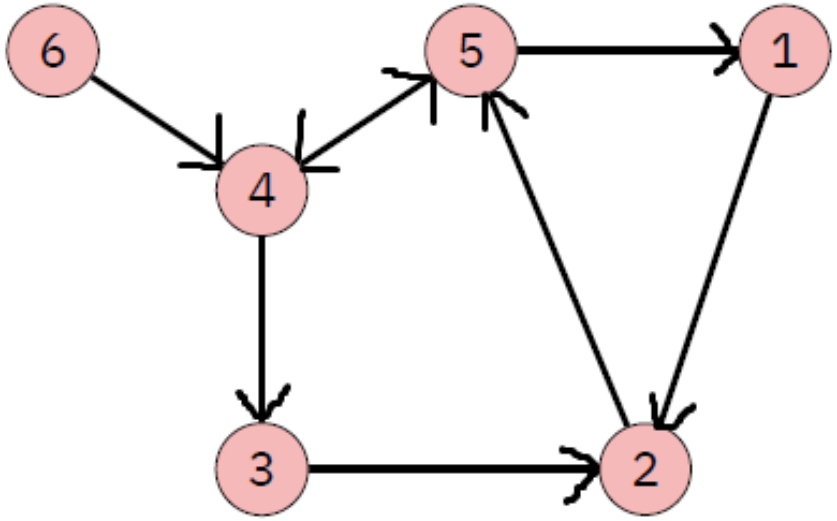


Graph theory – directed graphs

A **directed graph** is a pair G = (V, A) where A is the set of arcs, which are directed edges. We denote an arc from node i to node j by (i, j).



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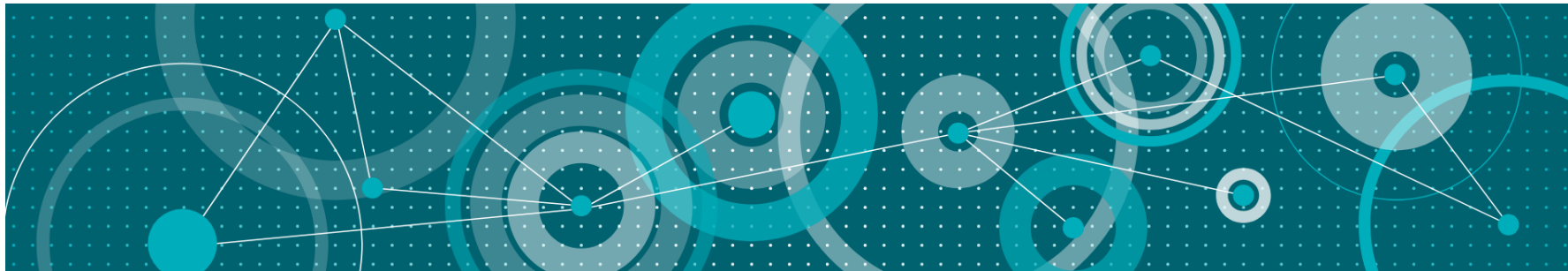
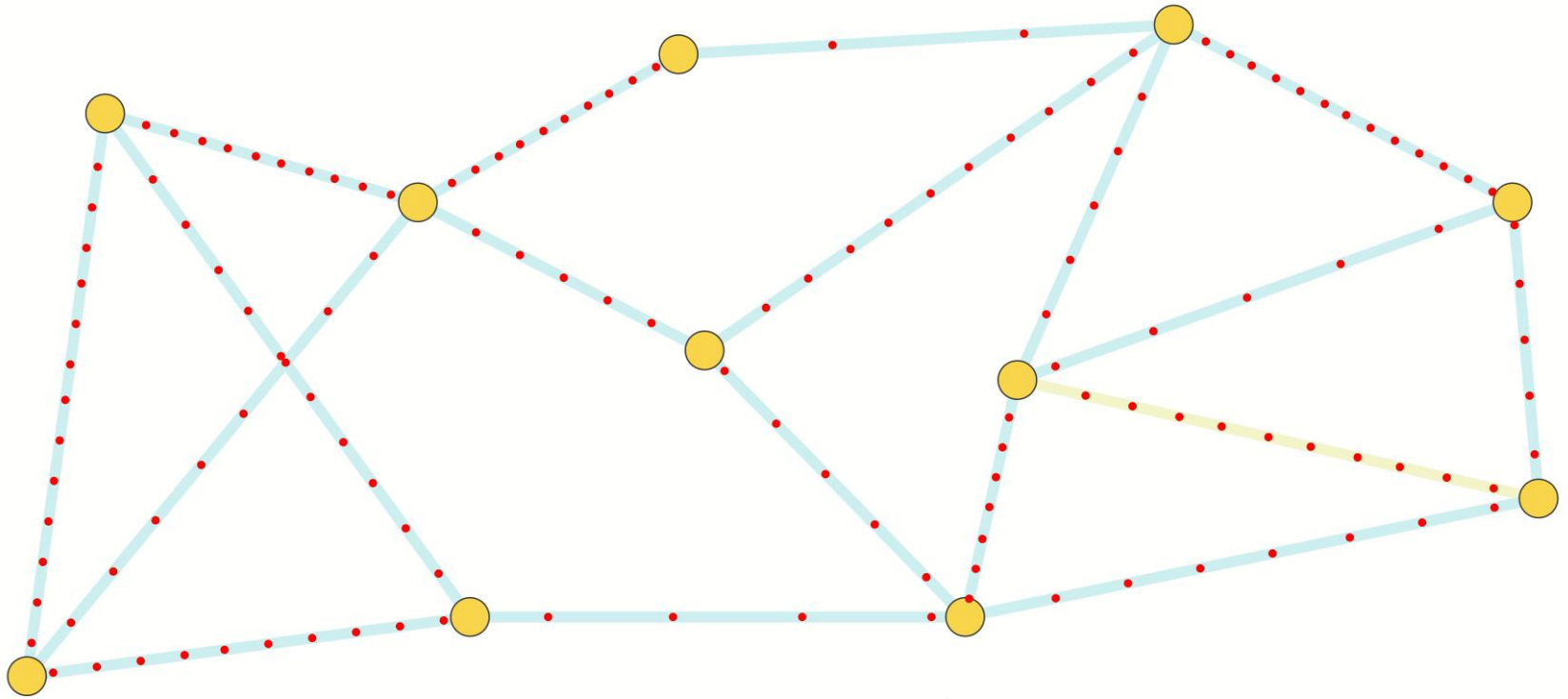


Graph theory – flows



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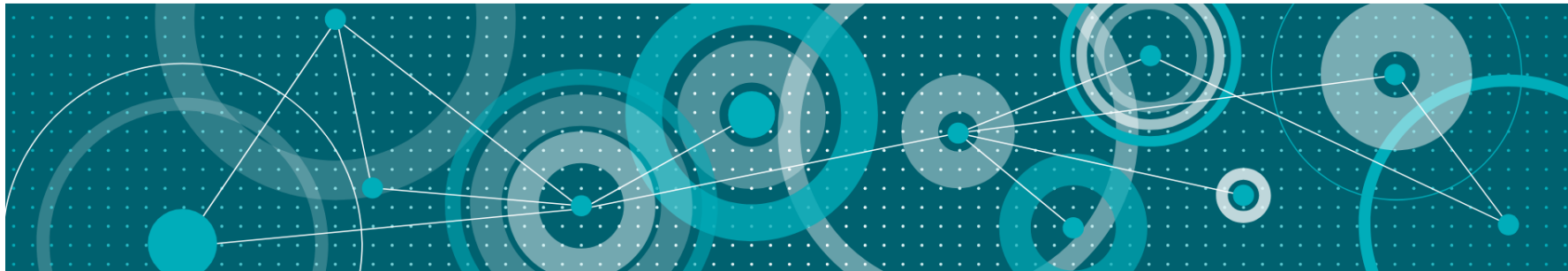
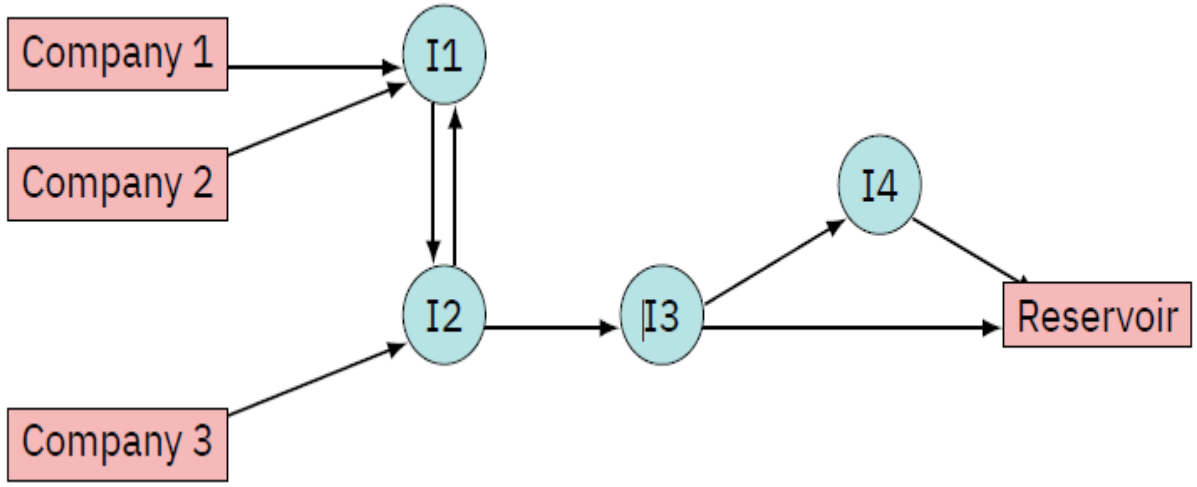


Graph theory – flows

Think of sewage water going from companies, through the intersections which are the nodes of the graph, to the reservoir, where the sewage water is purified.

In general, we will use the notation **s-t-flow** for a flow on a path in the graph starting from a source s and ending in a sink t.

What do you think? If you have such a flow what should hold for each node on the network?



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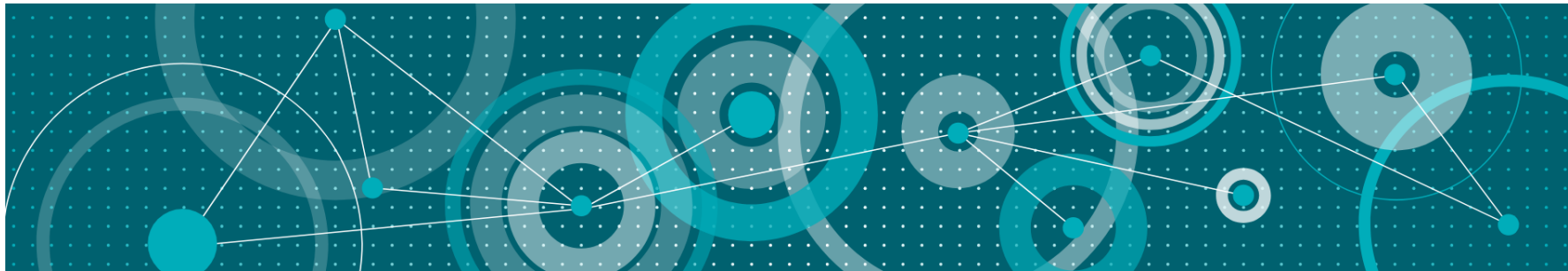


Graph theory – s-t flows

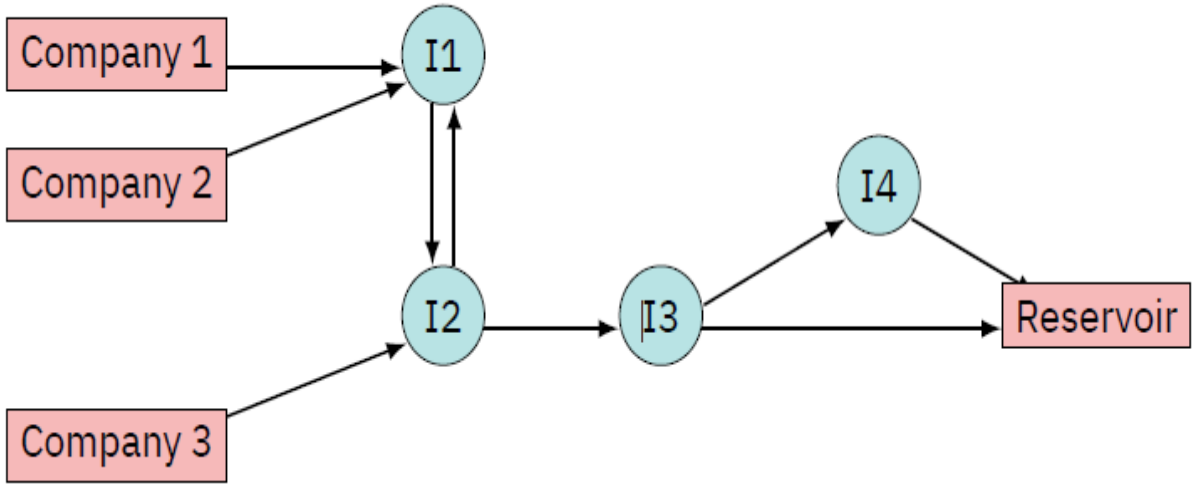
Let s and t be nodes in a directed graph G = (V;A).

Then an s-t flow is a collection of values f(a) on the arcs that defines a flow value for every arc such that flow conservation holds.

This means:

• For every node v (except s and t), the total flow value going into v equals the total flow value going out of v.

For simplicity, we assume s has only outgoing arcs, and t has only ingoing arcs. The ***value*** of an s -t flow is the total flow going out of s.



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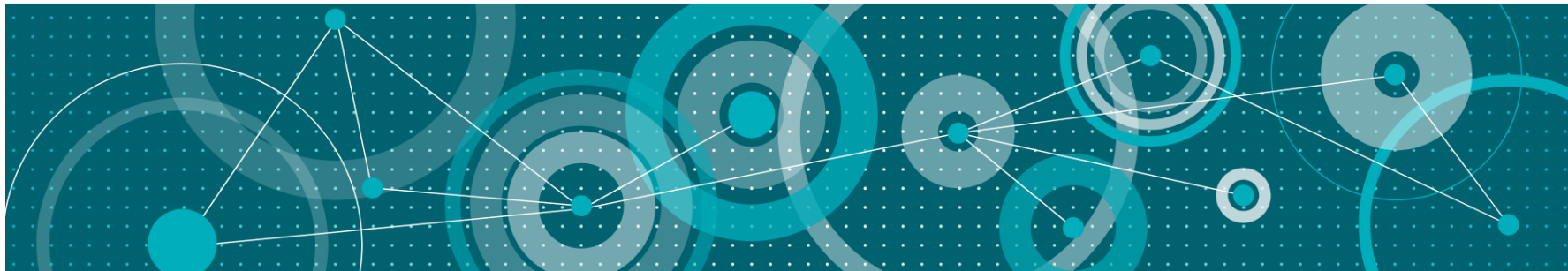
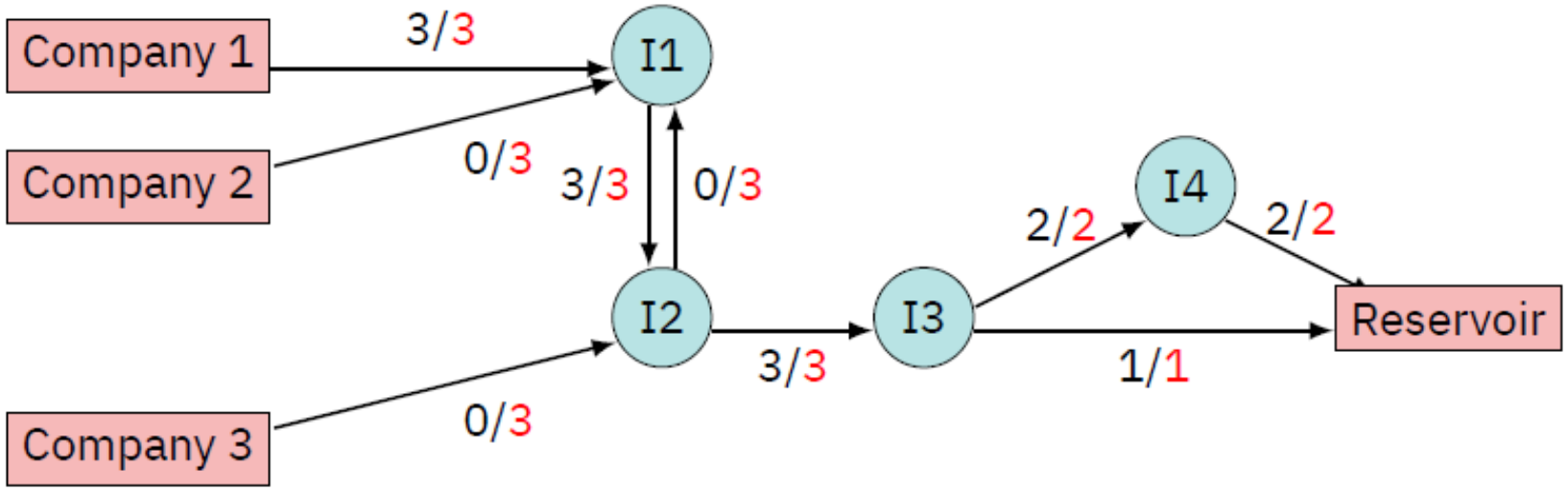
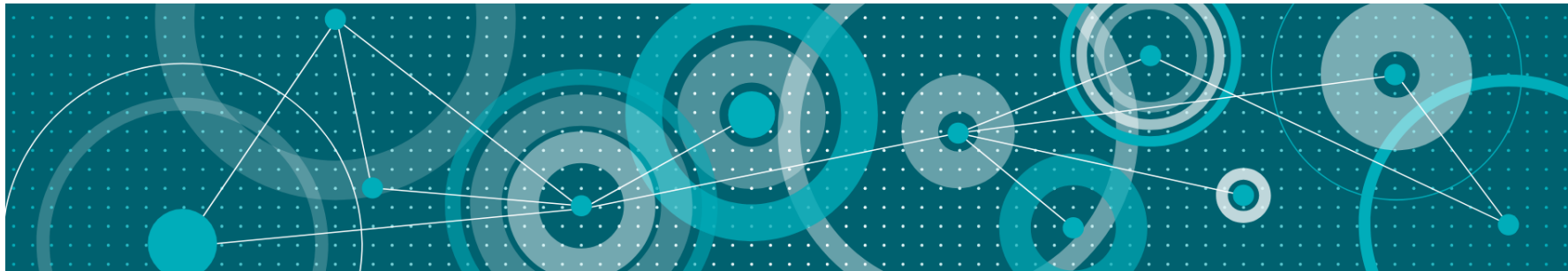


Graph theory – s-t flows

Flows usually become interesting when the arcs each have a certain capacity.

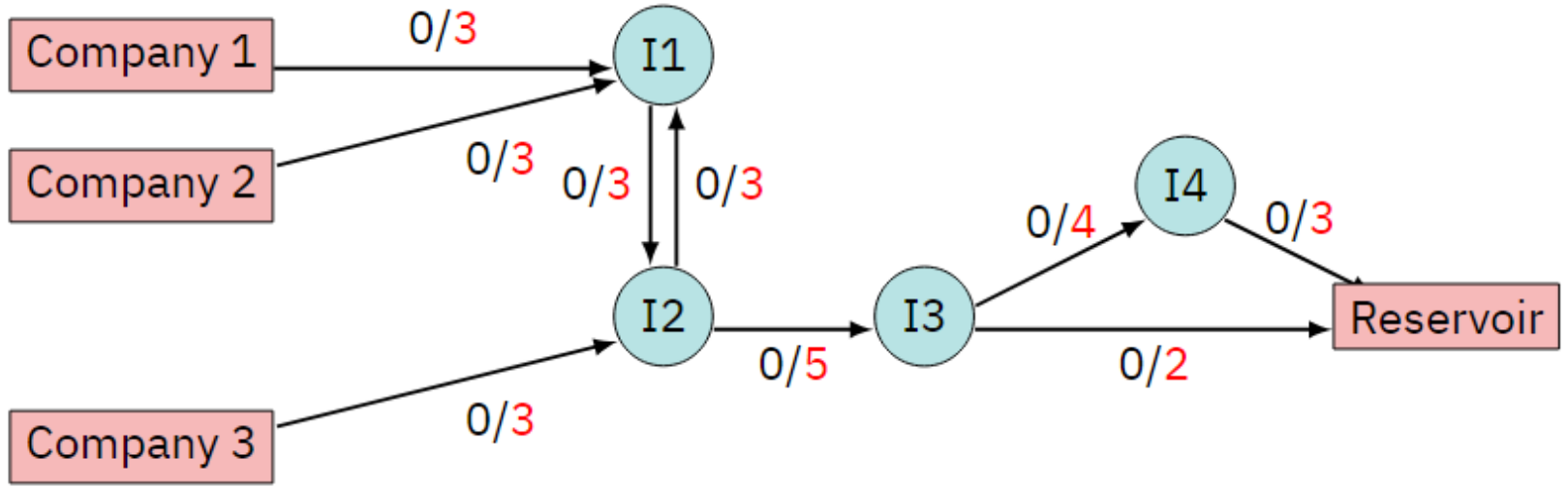
Denote the capacity of an arc a by c_a.

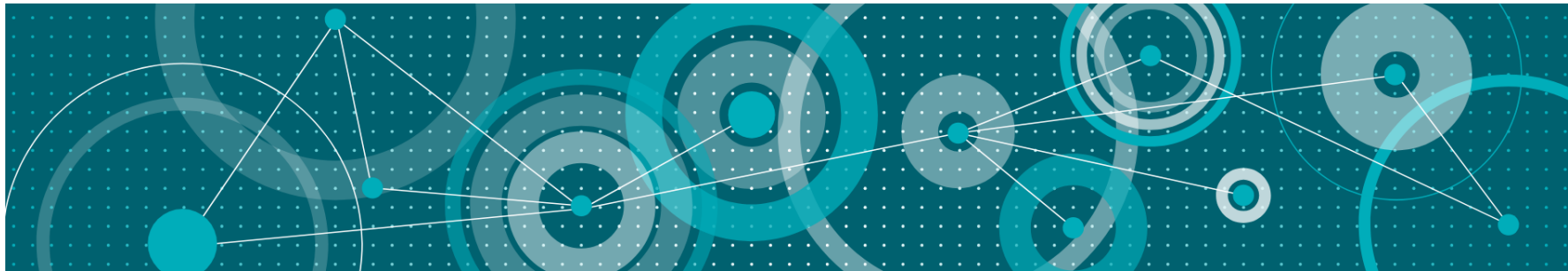




Graph theory – s-t flows

Exercise: find the maximum flow you can send to the Reservoir respecting the capacities of the arcs.





Game theory

How to make decisions

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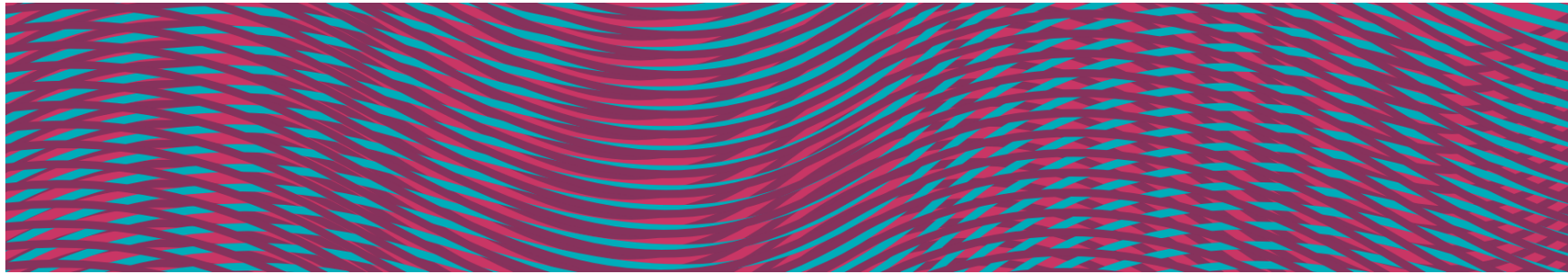
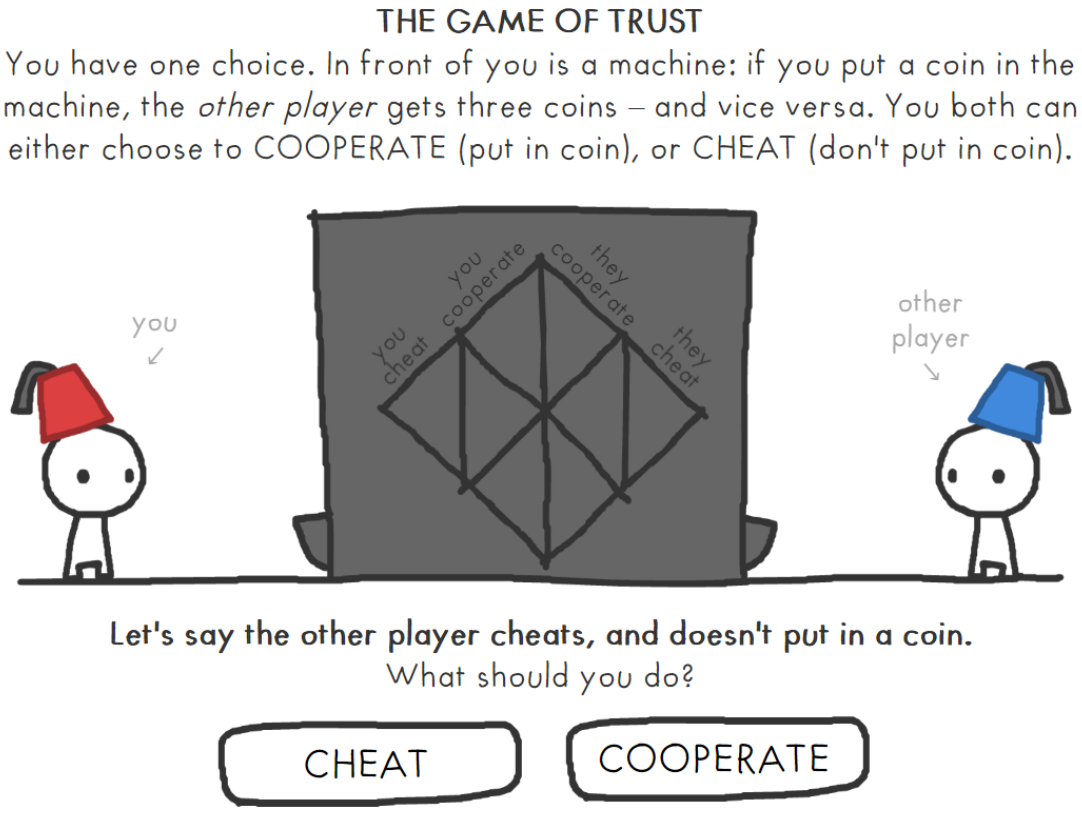


Game theory

Game theory is the study of mathematical models of strategic interactions.

In general, you have a situation where participants need to make a decision.

Each decision results in a profit or a penalty, and you want to understand how all the parties involved should behave if you want an optimal decision to be made.



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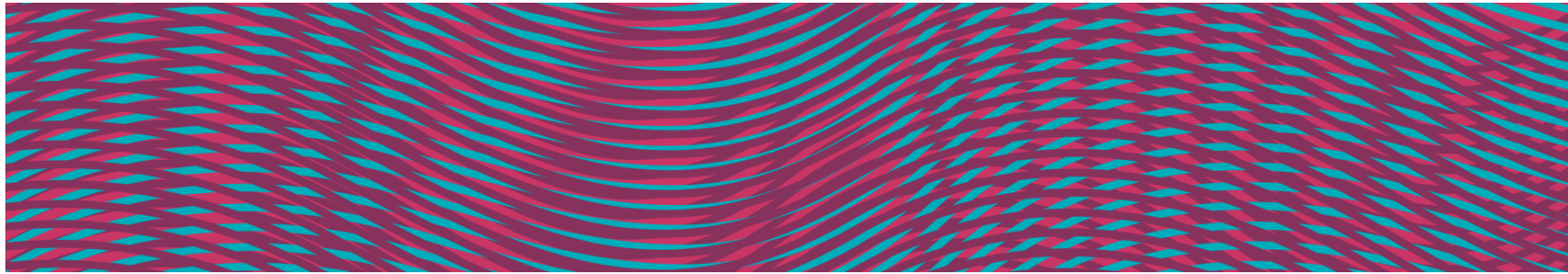

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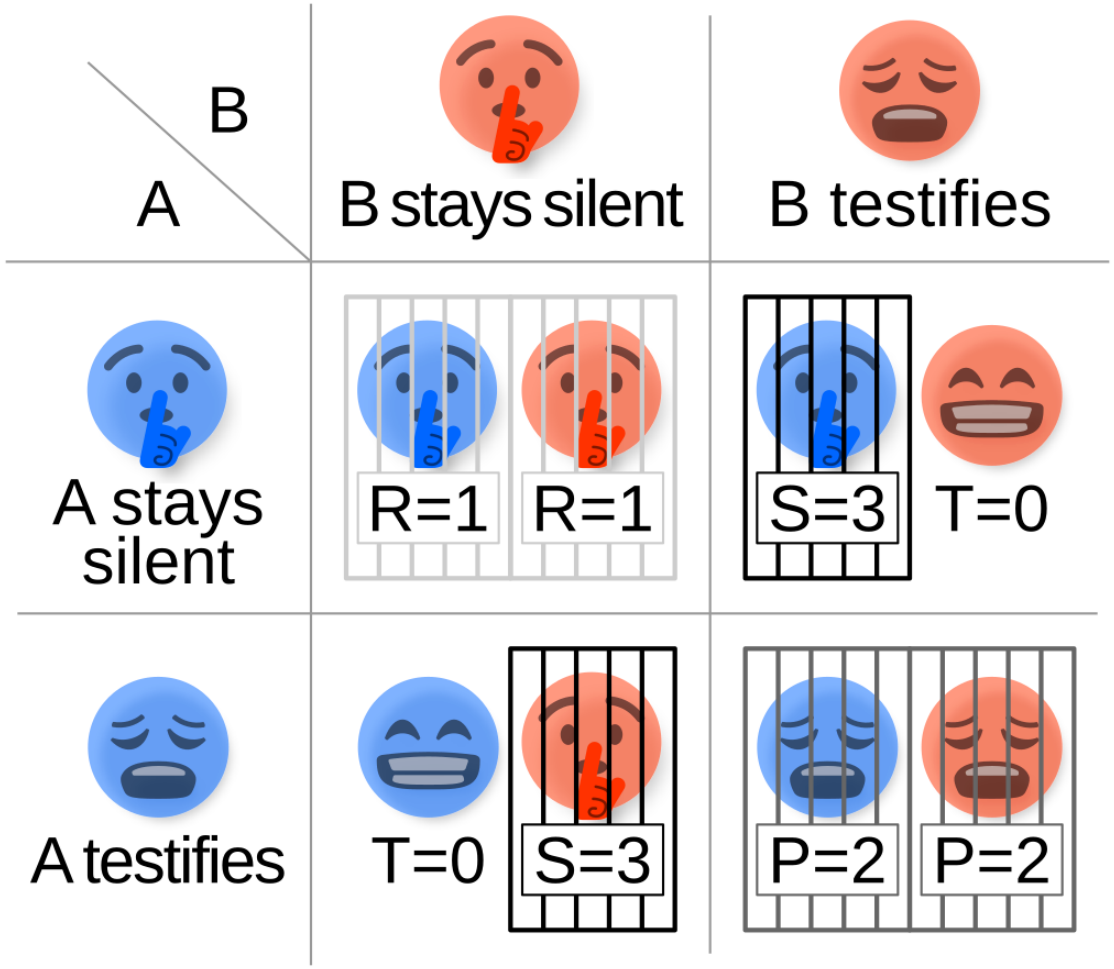
How should the parties involved behave if you want an optimal decision to be made?



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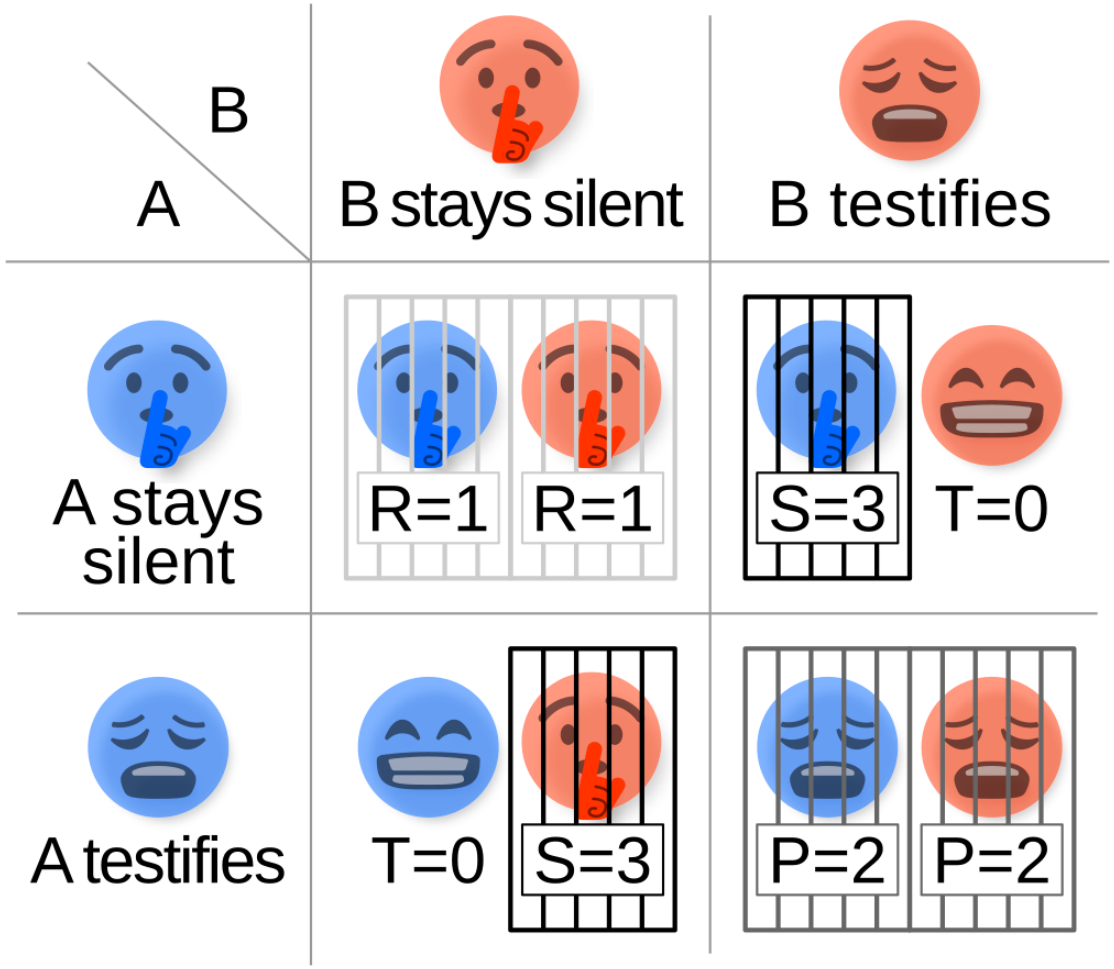
Game theory – Nash Equilibrium

A Nash equilibrium is a situation where no player could gain by changing their own strategy, holding all other players' strategies fixed.



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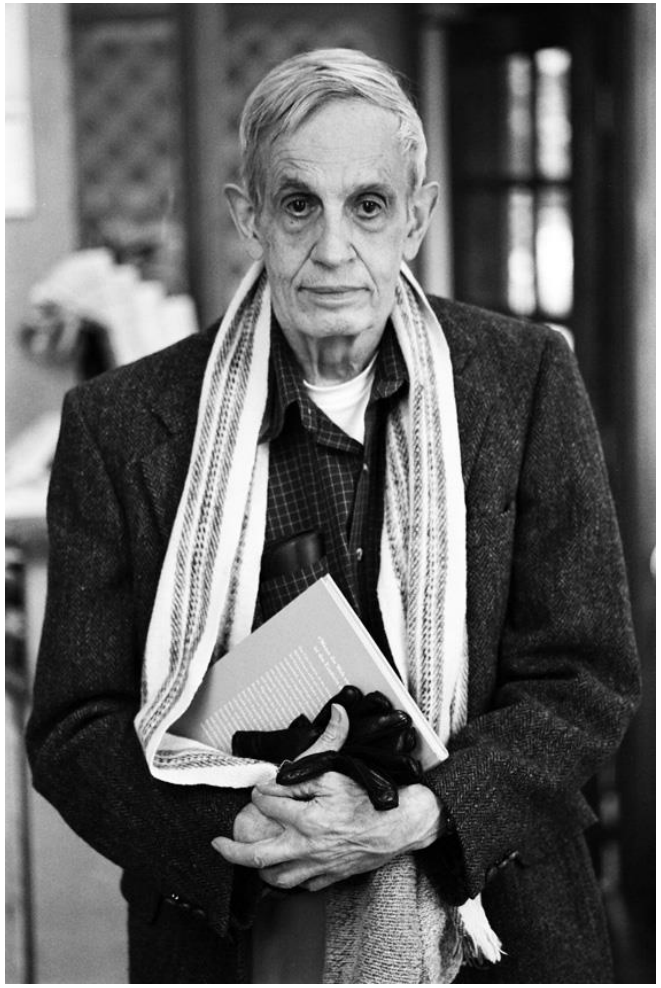
Game theory – Nash Equilibrium

Nobel prize in economics in 1994

Abel prize in 2015



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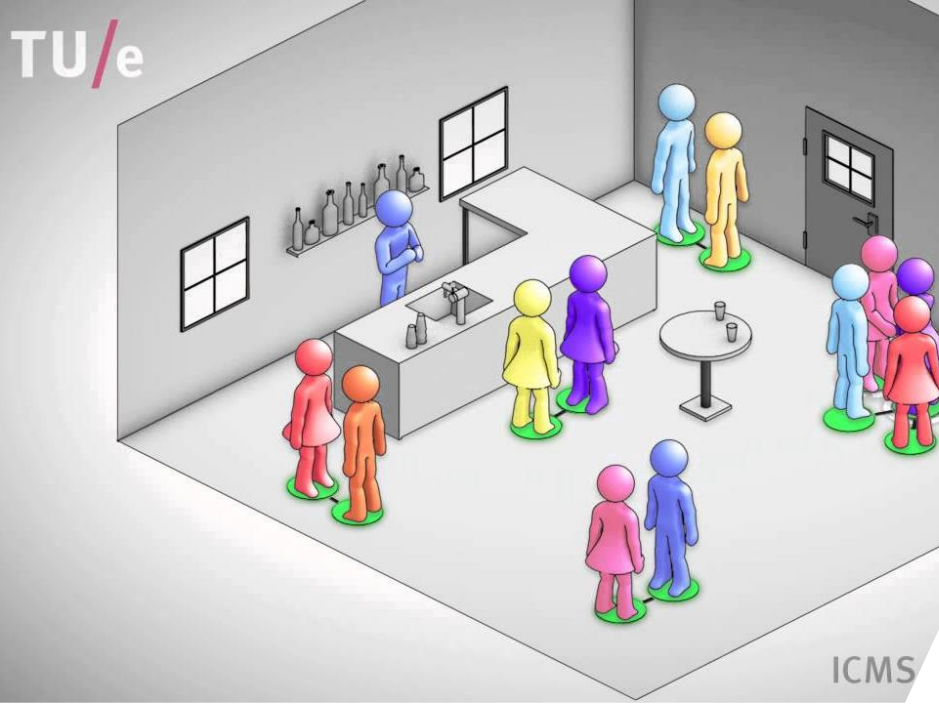
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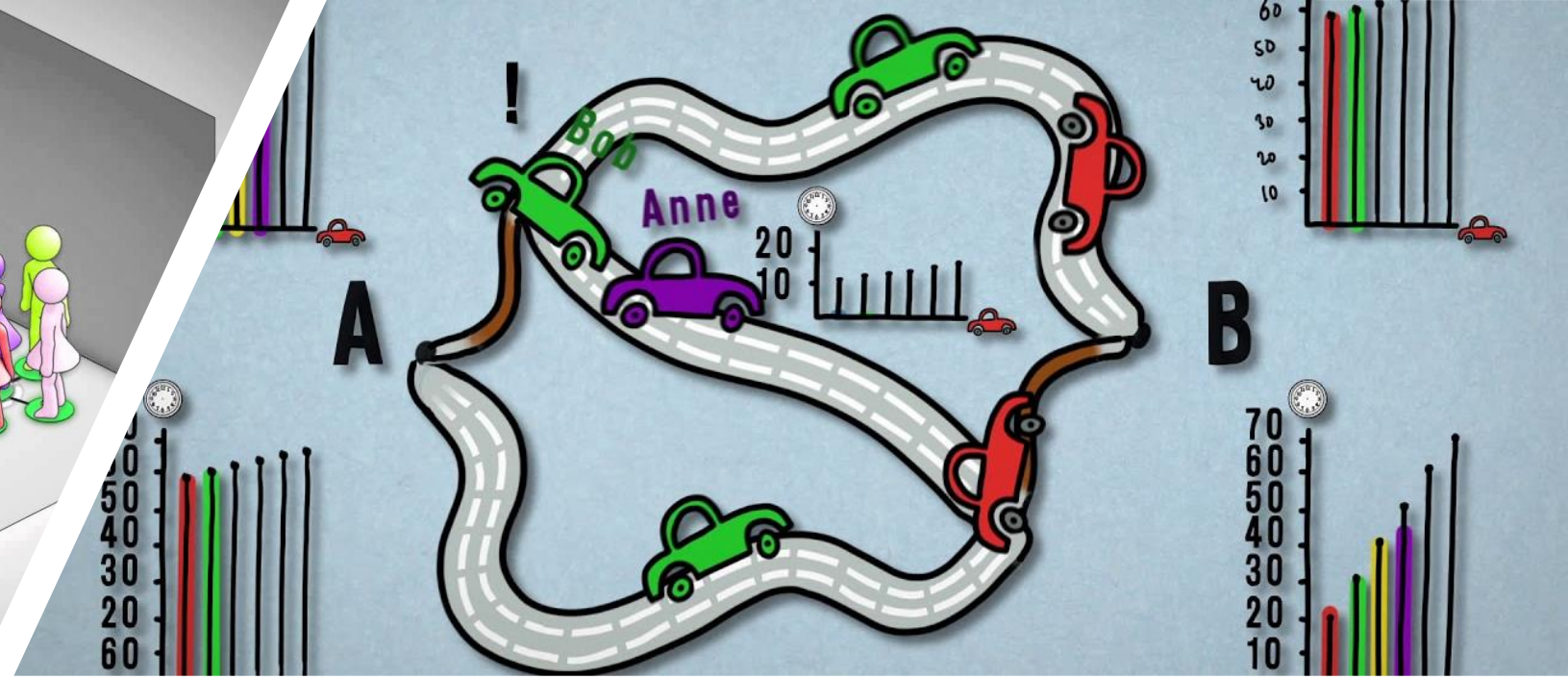
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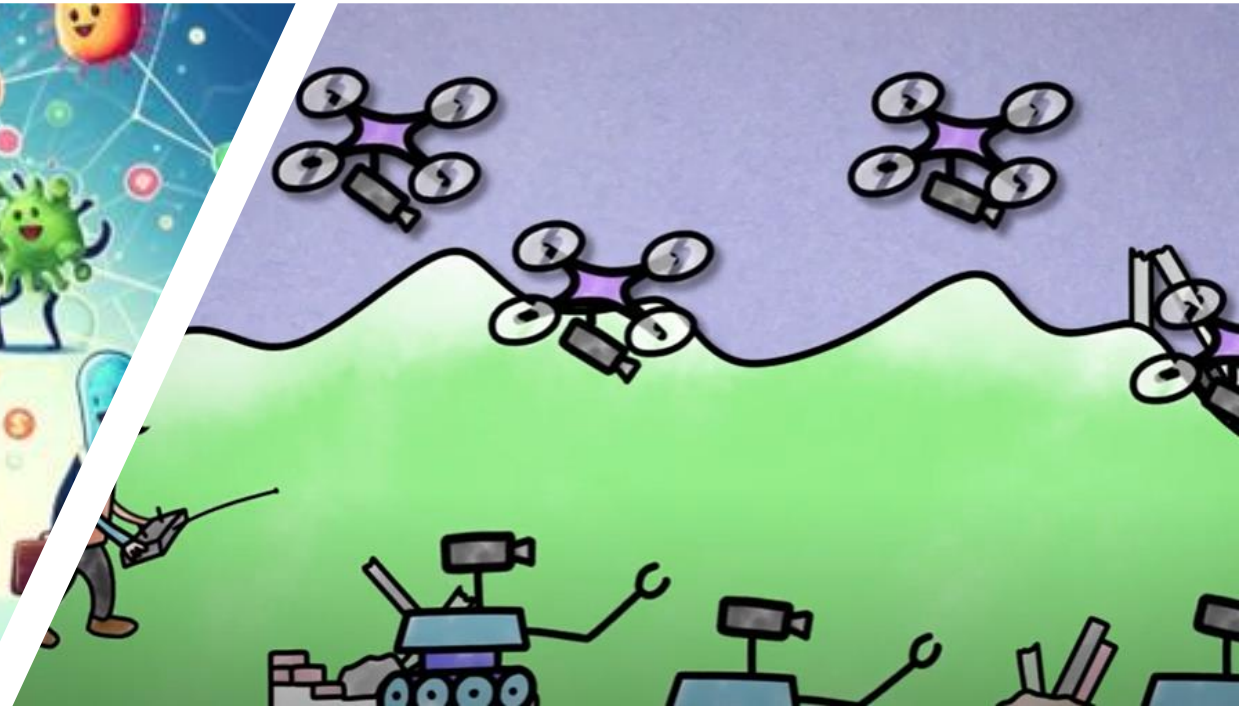

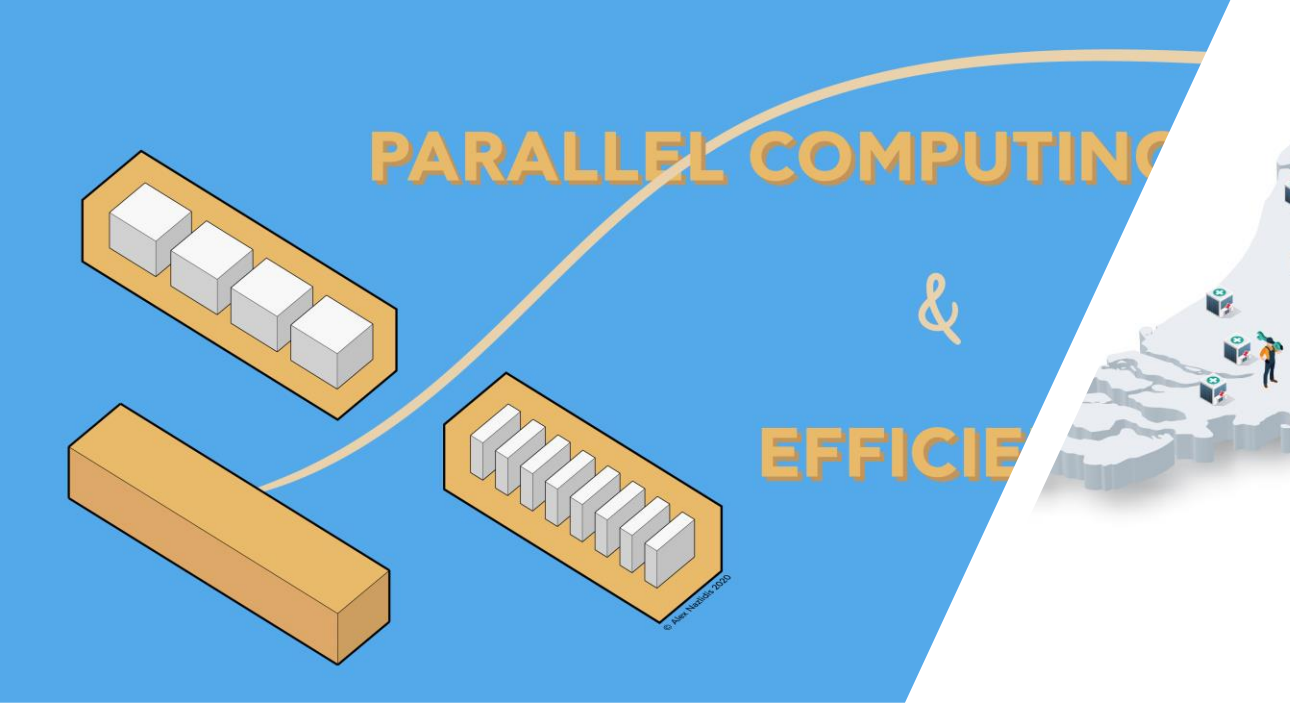
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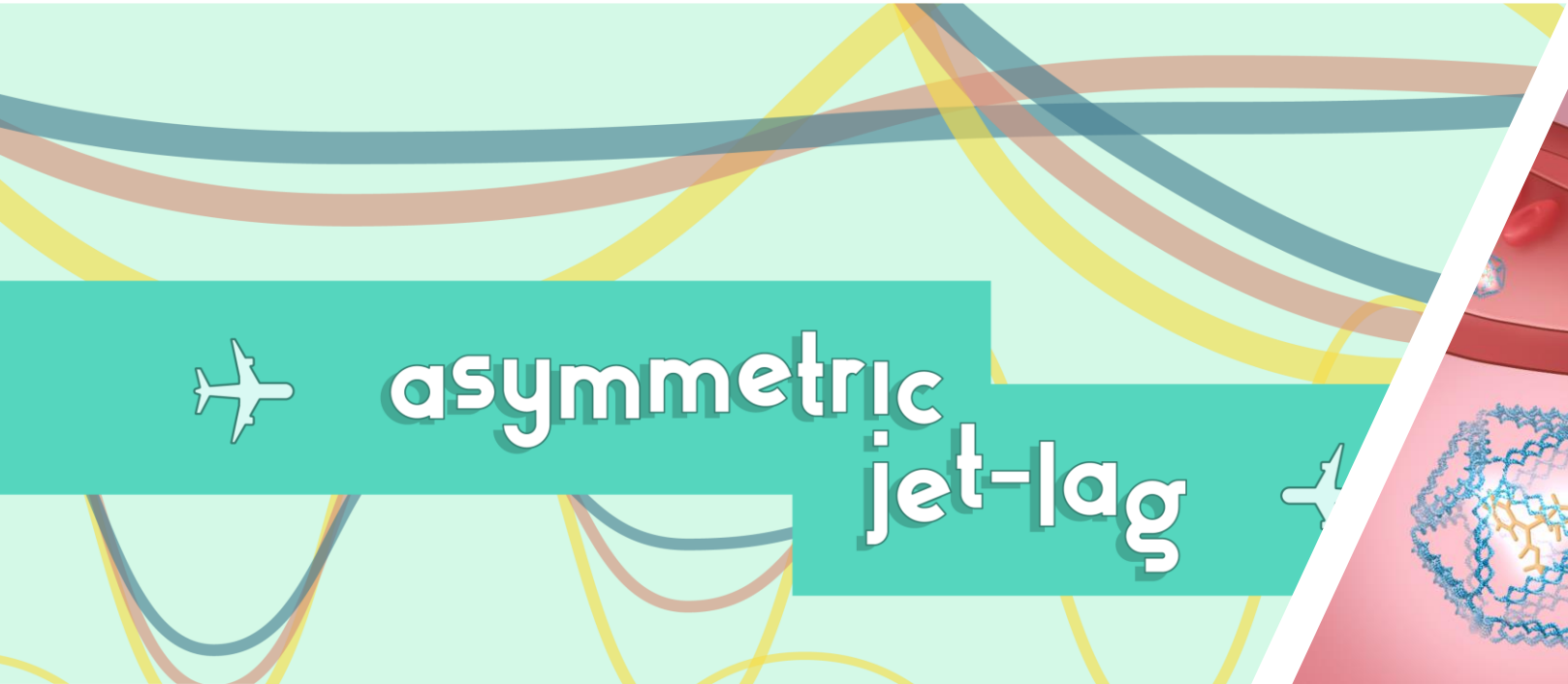
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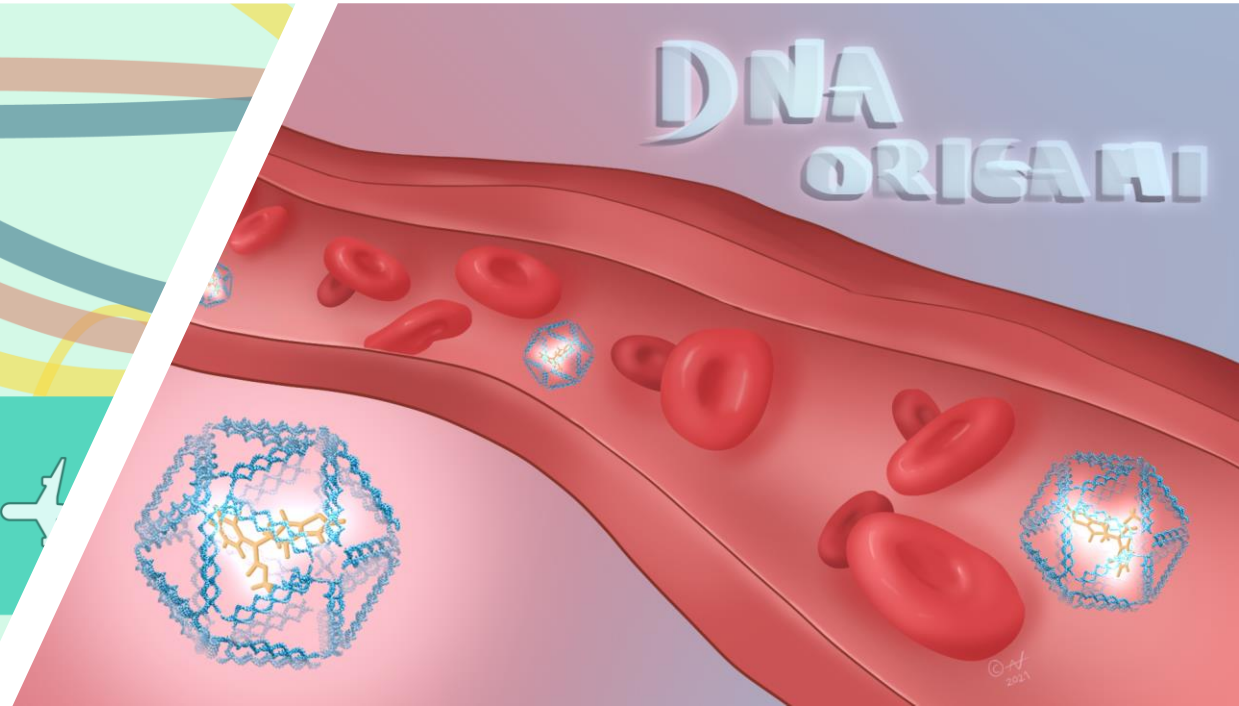
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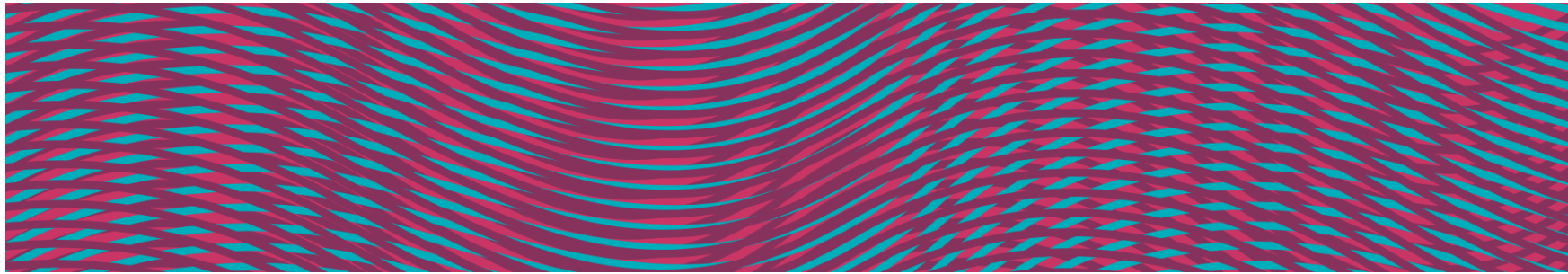

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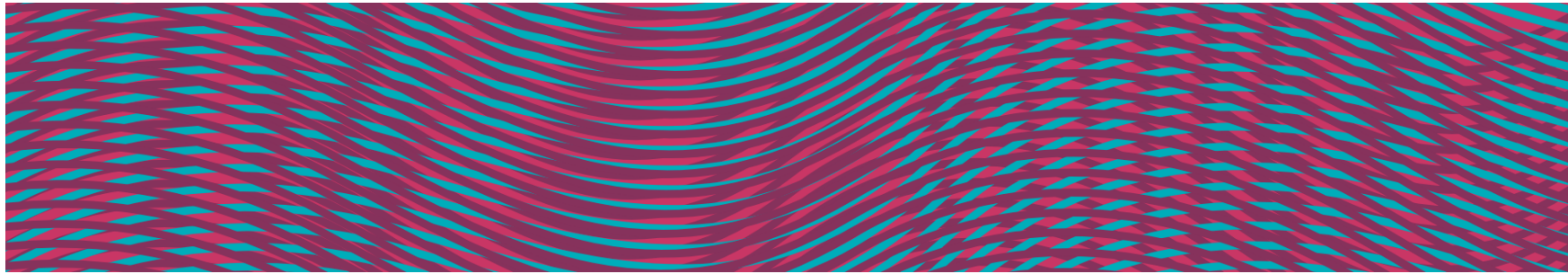

What is there for you?

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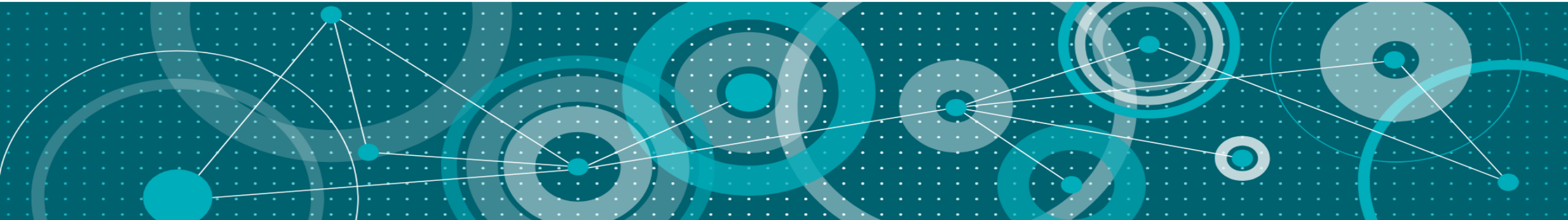
Network Pages: https://www.networkpages.nl/
Educational material (Dutch): www.onderwijs.networkpages.nl

On both websites you can subscribe to a **newsletter**!

Contact: editor@networkpages.nl, n.j.starreveld@uva.nl



A Dutch version for school pupils

Webclass on Complex Networks

Lecture notes on various topics

- ✓ Algorithms and Complexity
- ✓ The mathematics behind AI
- ✓ The mathematics behind Enigma
- ✓ The theorem of Descartes
- ✓ De Bruijn graphs and magic tricks
- ✓ Colorings of graphs
- ✓ Programming in Python for school students